The International Maritime Transport and Logistics (MARLOG 13) - ISSN 2974-3141
http://dx.doi.org/10.21622/MARLOG.2024.13.1.102
Towards Smart Green Blue Infrastructure

Pilot-Assisted Underwater Acoustic Channel Estimation for MIMO OFDM Systems Using Sparse Bayesian Learning Algorithm

Asmaa Yahia [1], and Eman Hamdan [2]

[1] Electrical Engineering Department, Faculty of Engineering Alexandria University, Alexandria, Egypt, asmaa.yahia22@alexu.edu.eg
[2] Marine Engineering Technology Department, College of Maritime Transport and Technology, AAST, eman_youssef@aast.edu

ABSTRACT

The field of underwater acoustic communication (UWA) has many industrial and maritime applications. This study focuses on cutting-edge channel estimation algorithms for UWA communications based on compressed sensing (CS). Since underwater channels involve sparse multipath, this investigation scrutinizes the process of channel estimation in systems employing multiple-input multiple-output (MIMO) technology with orthogonal frequency division multiplexing (OFDM). It interprets the utilization of pilot tones within the framework of a compressive sensing challenge. The performance of Compressive Sampling Matching Pursuit (CoSaMP) and Sparse Bayesian Learning (SBL) algorithms is compared with the conventional least square (LS) estimation algorithm by simulation.

The research infers that, methodologies rooted in compressed sensing yield superior channel estimation compared to the conventional LS algorithm for underwater communication systems utilizing MIMO-OFDM. For CS algorithms the simulation shows that SBL algorithm outperforms CoSaMP algorithm. Mean square error (MSE) and bit error rate (BER) are used to quantify this superiority when signal-to-noise ratio (SNR) conditions vary, employing both uniform and dispersed pilot configurations.

Keywords: channel estimation, sparse signal, Sparse Bayesian Learning, compressed sensing.

INTRODUCTION

Underwater communication systems play a pivotal role in various applications such as environmental monitoring, underwater surveillance, autonomous underwater vehicles (AUVs), and inspection of oil and gas pipelines(Dev Pratap Singh and Deepak Batham 2022). Unique challenges are encountered in the underwater acoustic (UWA) channel, such as long propagation delays, severe multipath effects, limited bandwidth, and frequency-dependent attenuation. Advanced techniques for accurate channel estimation and signal recovery are necessary to address these challenges, which significantly impact the reliability and data rate of communication systems(Khan, Das, and Pati 2020). Due to the use of multiple narrowband subcarriers, Orthogonal Frequency Division Multiplexing (OFDM) has shown to be an effective modulation system for underwater communication, reducing multipath fading and inter symbol interference (ISI). To improve the performance of OFDM in underwater environments, Pilot-based channel estimate is a well-established approach in which known symbols, or pilots, are embedded in the transmitted signal to facilitate accurate estimation of the channel response.

The application of Multiple Input, Multiple Output (MIMO) techniques to underwater communication systems holds the potential to revolutionize the way we perceive and design underwater communication networks as it offers increased data rates, improved reliability, and enhanced spectral efficiency(Altabbaa 2021). MIMO exploits spatial dimensions to enhance communication by transmitting multiple independent data streams simultaneously(Li et al. 2023). In underwater acoustic communication, spatial multiplexing involves the use of multiple transducers at both the transmitter and receiver.

In recent years, Compressed Sensing (CS) has garnered considerable attention as a powerful signal processing paradigm for sparse signal recovery. CS enables the reconstruction of sparse signals from a reduced set of measurements, providing an efficient and accurate alternative to traditional methods such as least square (LS). Comparative studies(Mechery and Remadevi 2017; Khan, Das, and Pati 2020) have evaluated the performance of compressed sensing-based channel estimation against traditional methods in underwater acoustic communication scenarios, providing insights into the advantages and limitations of CS.

Within the realm of underwater acoustic channel estimation, the integration of pilot-oriented MIMO-OFDM and compressed sensing enhances the performance metrics of mean square error (MSE) and bit error rate (BER). The improvement is achieved by decreasing the complexity of channel tracking and lowering the cost of hardware across various system model environments..(Khan, Das, and Pati 2020). By exploiting the sparsity inherent in the underwater channel, compressed sensing enables accurate estimation even in scenarios with limited resources or rapidly changing channel conditions.

This study presents a comprehensive comparison between the conventional LS algorithm for channel estimation and two prominent CS algorithms: Compressive Sampling Matching Pursuit (CoSaMP) and Sparse Bayesian Learning (SBL). CoSaMP is a greedy pursuit algorithm known for its simplicity and efficiency, while SBL leverages a probabilistic framework for sparse signal estimation.

The structure of this paper is as follows. First the model of the MIMO-OFDM communication system is elucidated. An explanation for the UWA channel is presented in Section 4. Section 5 is dedicated to the discussion of channel estimation. Simulation results are encapsulated in Section 6. Finally, Section 7 consolidates the conclusions.

MIMO SPATIAL MULTIPLEXING SYSTEM

For acoustic communications, OFDM is a low-complexity substitute for conventional single-carrier modulation. We advocate for a MIMO spatial multiplexing system to augment the data rate within a restricted acoustic bandwidth. A diagrammatic representation of the UWA OFDM transmitter is illustrated in Figure 1. Quadrature amplitude modulation (QAM) is employed to map and encode the binary data stream. Modulated signal undergoes a transformation from serial to parallel, with the inclusion of pilot tones for determining the channel impulse response (CIR). The UWA MIMO-OFDM system manipulates parallel data employing the inverse fast Fourier transform (IFFT).

$$x(n) = \mathrm{IFFT}\{X(k)\}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}nk}, \qquad n = 0, 1, \ldots, N-1 \tag{1}$$

Where x(n) and X(K) are the time domain signal and the frequency domain signal respectively.

Post-IFFT, the transformation of the N parallel subcarriers into a serial bit stream is executed, incorporating guard intervals made of cyclic prefix (CP) samples to avert ISI. Observationally, the final

Ng samples of $x(n)$ are duplicated as a CP and positioned at the commencement of this symbol, culminating in the signal $x(n)$ with a length equivalent to $N + Ng$, where Ng denotes the length of the CP samples. The signal $yg(n)$ is received subsequent to its transmission through the UWA channel.

$$yg(n) = xg \otimes h(n) + w(n), -Ng \leq n \leq N-1 \quad (2)$$

Herein, $\otimes$ denotes the circular convolution operator, while $w(n)$ signifies the additive white Gaussian noise (AWGN) with a zero mean, and $h(n)$ symbolizes the CIR. The received signal undergoes division into parallel subcarriers, and the CP is removed in the receiver, as illustrated in Figure 2. Fast Fourier transform (FFT) operations are employed to convert the time-domain waveform $y(n)$ into the frequency-domain waveform $Y(k)$, as detailed below:

$$Y(k) = \text{FFT}\{y(n)\}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} y(n)e^{-j\frac{2\pi}{N}nk}, \qquad k = 0, 1, \dots, N-1 \quad (3)$$

Afterward, the transformed signal is captured as a sequence and decoded by the appropriate transmitter algorithms after channel estimation. In this step, the UWA MIMO-OFDM system model's output is used to obtain the final binary data stream.

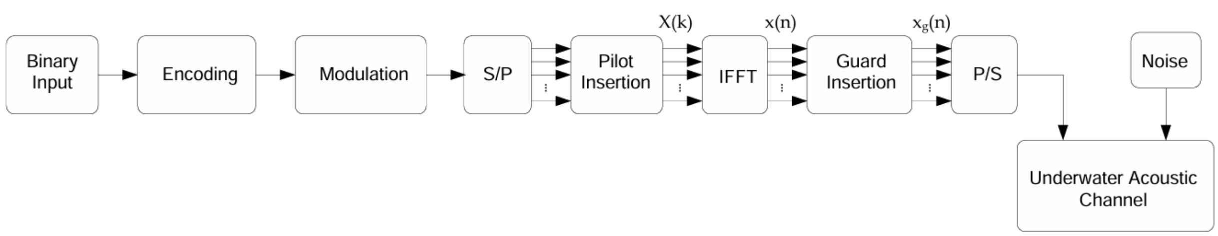


Figure 1: UWA communication system transmitter.

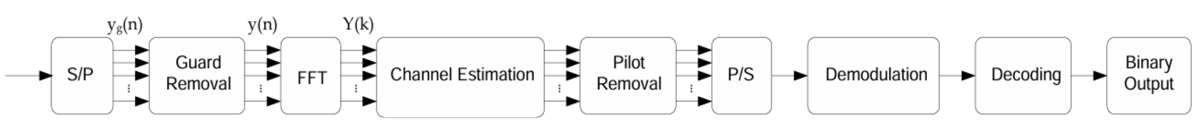


Figure 2: UWA communication system receiver.

UNDERWATER ACOUSTIC COMMUNICATIONS CHANNEL MODEL

The distribution of each channel gain may be deduced to be distinct based on the condition of the sea. In scenarios where the receiver is in close proximity to the transmitter in shallow water, the impact of diffuse random multipath contributions is considered insignificant, and the gains from channel taps are hypothesized to adhere to the Rician distribution. However, when the transmitter and receiver go farther apart, large sea dynamics obstruct direct route contributions, leading to a predominance of diffuse multipath and a Rayleigh distribution in the channel gains. We employ the channel transfer function pertinent to shallow UWA channels, which has been modeled and computed by the authors cited in (Qarabaqi and Stojanovic 2013). The UWA channel transfer function is expressed as follows:

$$H(f) = H_o \sum_{L} h_L \gamma_L(f) e^{-j2\pi\tau_L}, \quad (4)$$

$$\gamma_L(f) = \frac{1}{h_L} \sum_{i \geq 0} h_{L,i} e^{-j2\pi f \delta_{\tau_{L,i}}} \tag{5}$$

$H(f)$ denotes the channel transfer function, whereas $Ho(f)$ signifies the direct path transfer function. The small-scale fading coefficients are represented by $\gamma L(f)$, while hL and τL correspond to the path gain and delay, respectively. The intra-path gains and the propagation delay, associated with the L^{th} path, are symbolized by hL,i and $\tau L,i$, respectively.

CHANNEL ESTIMATION

Channel estimation employs pilot symbols, which are mutually recognized by both the transmitter and the receiver. Within the framework of OFDM frames, these pilot symbols can be allocated in the time domain, frequency domain, or both, resulting in various configurations such as comb-type, block-type, and scattered-type(Coleri et al. 2002; Barhumi, Leus, and Moonen 2003). Numerous interpolation methodologies can be employed to compute the channel responses of each subcarrier within the pilot symbols, subsequent to the state estimation at these pilot symbols. In this investigation, we utilize the pilot symbols at uniform intervals to scrutinize the optimal pilot sequence for UWA channel estimation and juxtapose it with the scattered configuration.

LS

Presume that the channel's sparsity level is denoted by $\boldsymbol{k}$ and the total count of taps is symbolized by $\boldsymbol{L}$. Given the sparse nature of the UWA channel, it implies that k is significantly less than L. The received vector can be articulated as follows:

$$Y = XFh + w \tag{6}$$

Where, $\boldsymbol{Y} = [\boldsymbol{Y}(\boldsymbol{0}), \boldsymbol{Y}(\boldsymbol{1}), \dots, \boldsymbol{Y}(\boldsymbol{L}-\boldsymbol{1})]^T$is the received signals after removing the CP, $\boldsymbol{X}$ is a $\boldsymbol{N} \times \boldsymbol{N}$ diagonal matrix of transmitted signal, includes the data matrix $\boldsymbol{D}$ and the pilot matrix $\boldsymbol{P}$, i.e., $\boldsymbol{X} = \boldsymbol{D} + \boldsymbol{P} = \boldsymbol{diag}[\boldsymbol{X}(\boldsymbol{0}), \boldsymbol{X}(\boldsymbol{1}), \dots, \boldsymbol{X}(\boldsymbol{N}-\boldsymbol{1})]^T$, $\boldsymbol{F}$ is $\boldsymbol{N} \times \boldsymbol{L}$ DFT matrix, the channel vector $\boldsymbol{h} = [\boldsymbol{h}(\boldsymbol{0}, \boldsymbol{h}(\boldsymbol{1}), \dots, \boldsymbol{h}(\boldsymbol{L}-\boldsymbol{1})]^T$, and $\boldsymbol{w} = [\boldsymbol{w}(\boldsymbol{0}), \boldsymbol{w}(\boldsymbol{1}), \dots, \boldsymbol{w}(\boldsymbol{L}-\boldsymbol{1})]^T$ is the noise vector which obeys a Gaussian distribution with zero-mean. The received pilot symbol samples are utilized for the channel estimate, which can be stated as follows:

$$Y_p = PF_p h + w_p = Ah + w_p$$

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N_p-1) \end{bmatrix} = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & P_{Np} \end{bmatrix} \cdot F_{Np \times L} \cdot \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(L-1) \end{bmatrix} + \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(N_p-1) \end{bmatrix} \tag{7}$$

Herein, Np denotes the pilot subcarriers. Conventionally, $A = PFp$ is identified as the sensing. P represents the $Np \times Np$ diagonal matrix of the pilot symbols. Fp is a $Np \times L$ matrix, constituted by the initial L columns of the DFT matrix F, and the Np rows of the selected matrix correlated with the pilot subcarriers, which can be articulated as(Shi and Yang 2016):

$$F_p = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \omega^{p_1} & \dots & \omega^{p_1(L-1)} \\ 1 & \omega^{p_2} & \dots & \omega^{p_2(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{p_{Np}} & \dots & \omega^{p_{Np}(L-1)} \end{bmatrix} \tag{8}$$

where $\omega = e^{-j2\pi/N}$. According to (Shi and Yang 2016), the LS algorithm offers the following solution for channel estimation, assuming that the estimated channel impulse response is $\hat{H}$:

$$\widehat{H}_{LS} = (A^H A)^{-1} A^H . y \tag{9}$$

Compressed sensing algorithm

A multitude of algorithms have been explored in (Khan, Das, and Pati 2020; Yahia, Alim, and Korany 2023) for the purpose of estimating the CIR of the UWA channel. A substantial number of pilots are necessitated for channel estimation, given that LS is profoundly influenced by noise and fails to account for the sparseness of the UWA channel. Furthermore, compressed sensing (CS) methodologies are contemplated for addressing these issues. CS employs a limited quantity of measurements, fewer than the signal dimensions, to identify a compressive representation of a signal (Mechery and Remadevi 2017; Wu et al. 2020; Jiang et al. 2018). As a result, the ensuing underdetermined linear system can be resolved:

$$y = Ax \tag{10}$$

In this case, $y \in \mathbb{R}^m$is the measurement vector, A is a $m \times n$ which is called sensing matrix with $m < n$, and $x \in \mathbb{R}^n$ is the signal we seek to identify, In compressed sensing and related fields, It is widely acknowledged that if the signal x possesses a sparse representation (a limited number of non-zero components), It can be distinctly reinstated (Mechery and Remadevi 2017)(Jiang et al. 2018). Finding the sparse solution for the underdetermined linear system requires solving the following optimization problem:

$$\min_x \|x\|_0 \text{ subject to } Ax = y \tag{11}$$

Herein, $\|x\|_0$ denotes the count of non-zero components in vector x, referred to as thel_0 − norm $\|x\|_0 \overset{\text{def}}{=} \#\{i : xi \neq 0\}$. After solving the optimization issue represented by (11), a sparse solution is obtained, which represents the given vector y as a linear combination of minimum number of columns of the measurement matrix A. relaxing the l_0 − norm to an l_1 − norm is a possible approach for obtaining a solution to (11) since that the l_0 − norm is a nondeterministic polynomial (NP)-hard problem which is a convex function, to ascertain the values of each coefficient xi for which the l_1 − norm $\overset{\text{def}}{=} \Sigma|xi|$ is minimized(Ramirez, Kreinovich, and Argaez 2013; Elad 2010; Tropp 2006; Natarajan 1995):

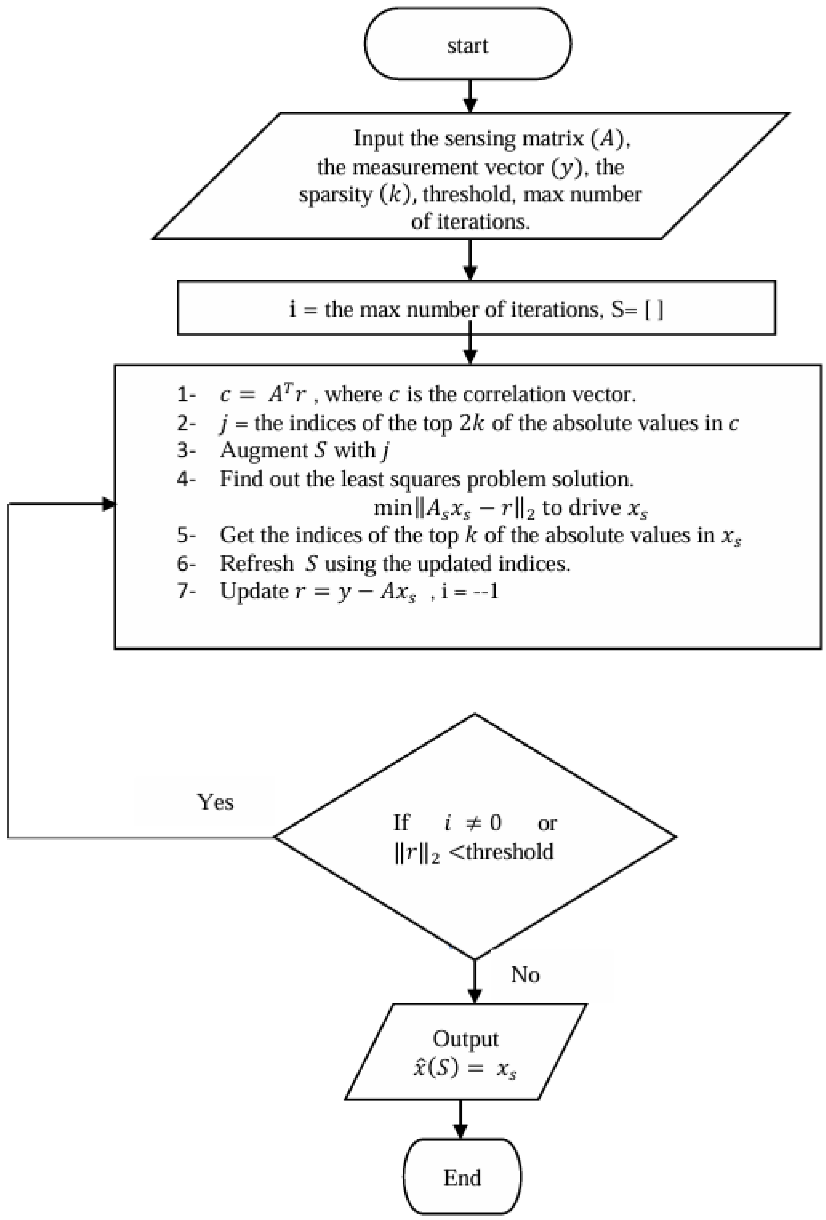
$$\min_x \|x\|_1 \text{ subject to } Ax = y \tag{12}$$

Employing greedy algorithms is another prevalent strategy for identifying a sparse solution to the underdetermined linear system as depicted in equation (10). These algorithms are efficacious and beneficial, and they find extensive application in compressed sensing due to their straightforward implementation and relatively modest computational demand Two prominent greedy algorithms utilized for compressed sensing are OMP and CoSaMP algorithms. Columns of the measurement matrix that show the strongest correlation with the residual signal are iteratively selected by the greedy OMP algorithm. CoSaMP is an OMP modification that augments performance by incorporating a thresholding step. The predicament with OMP and CoSaMP is that it may not be feasible to ascertain the sparseness degree k of the sparse vector x in advance while estimating sparse multipath channels (Aich and Palanisamy 2017; Rao and Kartheek 2018; Lu et al. 2019). To circumvent this issue, we employed the Sparse Bayesian Learning algorithm (SBL).

Compressive Sampling Matching Pursuit (CoSaMP)

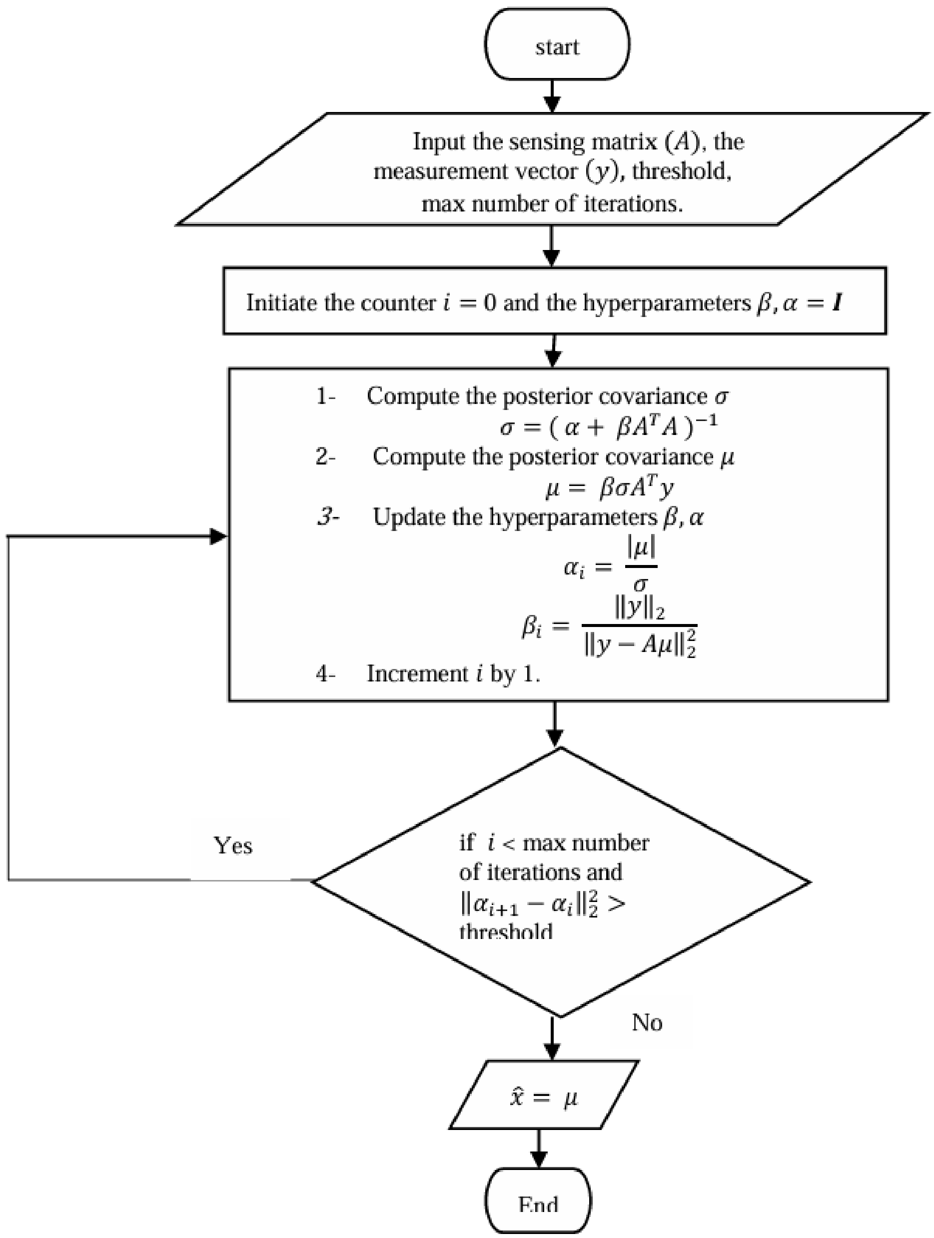
CoSaMP is predicated on OMP, discerns the support set S by computing the correlation between each measurement matrix A column and the residual vector r. Subsequently, these correlations are sorted in a descending sequence, and the top $2k$ indices j are selected and appended to S. Next, utilizing least-squares on the submatrix As, an estimate for the signal xs is calculated, this is created by choosing just the columns that match the indices in S. The largest k elements of $|xs|$ are utilized as a new xs .

Following this, the absolute values of xs are sorted in descending order, and the first k values are taken. To verify convergence, one can calculate the residual vector's norm. The algorithm reaches the maximum number of iterations or this norm decreases below a certain tolerance threshold, in which case the loop is broken(Lu et al. 2019; Needell and Tropp 2009). The CoSaMP algorithm according to (Aich and Palanisamy 2017) is elucidated as follows:



Sparse Bayesian Learning algorithm (SBL)

Sparse Bayesian learning is an effective method recently used for UWA channel estimation in CS(Qiao et al. 2018; Jia et al. 2023). It creates a flexible and reliable strategy that can adjust to the data by integrating the idea of sparsity with a Bayesian approach to learning(Yang, Xie, and Zhang 2012). Starting with a prior distribution that reflects our initial assumptions about the parameters, it uses the data that has been observed to update its distribution. The result is the posterior distribution, which represents our updated assumptions about the parameters. The sparsity of the representation is controlled by hyperparameters. In SBL, these hyperparameters are also learned from the data, which is a key advantage of the method. The learning algorithm in SBL involves iteratively updating the model parameters and the hyperparameters until convergence. This is typically done using an Expectation- Maximization (EM) algorithm(Ament and Gomes 2021). The Sparse Bayesian Learning (SBL) algorithm is elucidated in depth as follows:



SIMULATION AND RESULTS

Derived from the empirical data procured from the Kauai AComms Multidisciplinary University Research Initiative (MURI) (KAM), conducted proximate to the shoreline of Kauai Island, HI, USA, the carrier frequency was established at 13 kHz. The aquatic depth was measured at 100 m, and both the transmitter and the receiver were positioned at 58 and 59 meters above the seabed, respectively. The signals were intercepted by the receiver, situated 3 km distant from the transmitter(Qarabaqi and Stojanovic 2013). An approximate representation of the UWA CIR is exhibited in Figure 3. In the simulations, pilot carriers are employed to find the channel frequency response. Table 1 contains the simulation's variables.

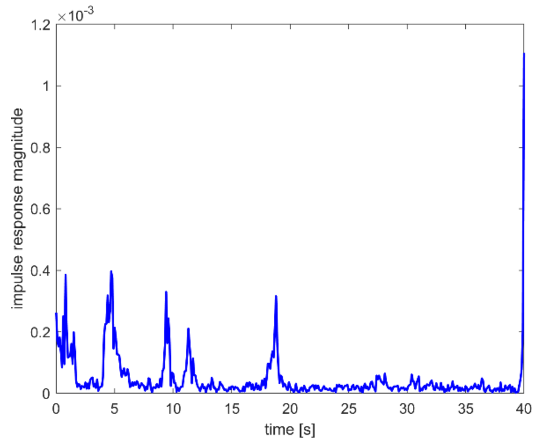


Figure 3: Channel impulse response of KAM experiment.

Table 1. Table 1 MIMO_OFDM Communication system parameters

Variable	Value	Variable	Value
FFT size	512	No. of transmitter antenna	2
CP length	128	No. of receiver antenna	4
Modulation	16-QAM	Channel model	MIMO
SNR, dB	0:5:30	Channel configuration	UWA-KAM

In Figures 4 and 5, a comparative analysis of the performance metrics, namely MSE and BER, is presented for standard LS, CoSaMP, and SBL under varying SNR conditions, utilizing a regular pilot arrangement. The analysis reveals a superior performance of SBL in comparison to both CoSaMP and LS.

Figure 6 elucidates the performance characteristics of the SBL algorithm, employing both regular and scattered pilot arrangements. Upon comparative evaluation, it becomes evident that the regular pilot arrangement exhibits a higher degree of compatibility with the SBL algorithm.

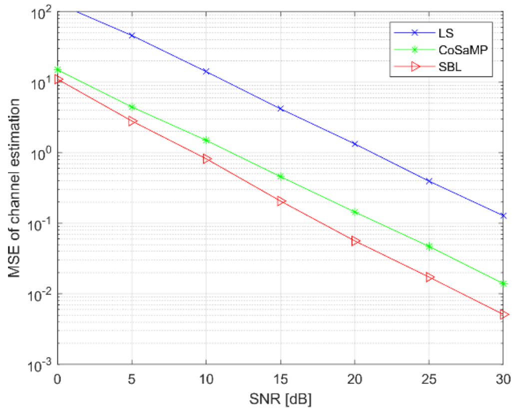


Figure 4: The performance of MSE for LS, CoSaMP, and SBL.

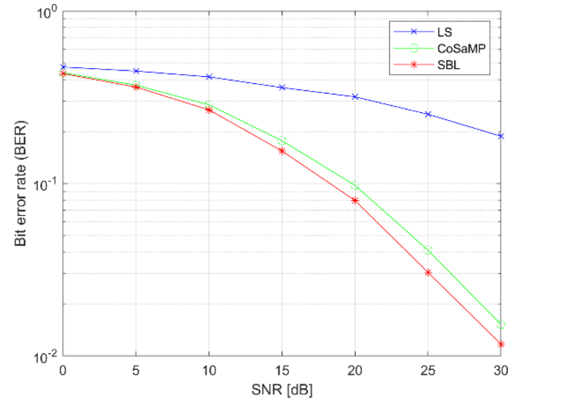


Figure 5: The performance of BER for LS, CoSaMP, and SBL

Given that the SBL and the LS algorithms do not necessitate channel sparsity, Figure 7 juxtaposes the original channel with the estimated channel derived from both SBL and LS algorithms. The comparative analysis indicates that SBL generates a sparse signal, predominantly characterized by zero values interspersed with occasional spikes. Conversely, LS disregards the channel's sparsity, resulting in a practical non-zero value for all its components. Consequently, the performance of SBL surpasses that of the LS estimate.

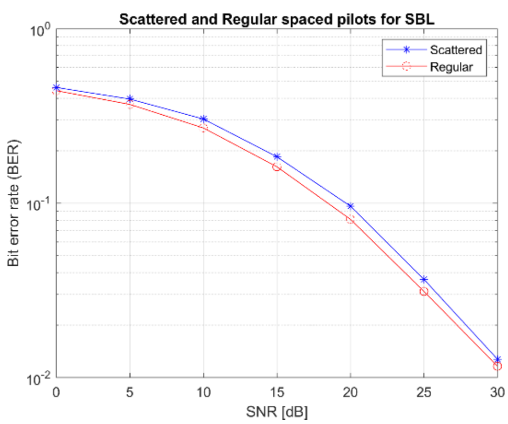


Figure 6: The pilot arrangement effect on the SBL algorithm.

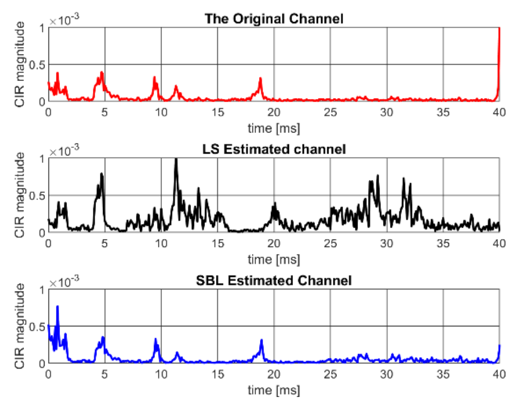


Figure 7: Channel Impulse Response.

CONCLUSIONS

This research undertakes the estimation of a pilot-assisted MIMO-OFDM-based underwater channel utilizing algorithms such as LS, CoSaMP, and SBL. A comparative performance analysis of these algorithms is conducted. The findings illustrate that the compressed sensing algorithms, CoSaMP and SBL, outperform the conventional LS method in the context of Underwater Acoustic channel estimation. However, CoSaMP's performance is contingent on the knowledge of the degree of sparsity, which is typically unavailable in most multipath channel scenarios, thereby positioning SBL as the optimal compressed sensing candidate. For the SBL algorithm, a regular pilot arrangement proves to be more efficacious for UWA channel estimation compared to a scattered arrangement. Future research endeavors will focus on further exploration of Bayesian-based compressed sensing algorithms for UWA channel estimation.

REFERENCES

[1] Aich, Abhishek, and P Palanisamy. 2017. "On Application of OMP and CoSaMP Algorithms for DOA Estimation Problem." In *2017 International Conference on Communication and Signal Processing (ICCSP)*, 1983–87. IEEE.

[2] Altabbaa, Mhd Tahssin. 2021. "Pilot Assisted MIMO-OFDM-IM Design for Time-Varying Underwater Acoustic Communications." *Wireless Personal Communications* 121 (1): 399–409.

[3] Ament, Sebastian E, and Carla P Gomes. 2021. "Sparse Bayesian Learning via Stepwise Regression." In *International Conference on Machine Learning*, 264–74. PMLR.

[4] Barhumi, Imad, Geert Leus, and Marc Moonen. 2003. "Optimal Training Design for MIMO OFDM Systems in Mobile Wireless Channels." *IEEE Transactions on Signal Processing* 51 (6): 1615–24.

[5] Coleri, Sinem, Mustafa Ergen, Anuj Puri, and Ahmad Bahai. 2002. "Channel Estimation Techniques Based on Pilot Arrangement in OFDM Systems." *IEEE Transactions on Broadcasting* 48 (3): 223–29.

[6] Dev Pratap Singh, and Deepak Batham. 2022. "A Review of Underwater Communication Systems." *International Journal of Engineering Development and Research* 10 (2): 100–104. www.ijedr.org.

[7] Elad, Michael. 2010. *Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing*. Vol. 2. Springer.

[8] Jia, Shuyang, Sichen Zou, Xiaochuan Zhang, and Lianglong Da. 2023. "Underwater Acoustic Channel Estimation Based on Sparse Bayesian Learning Algorithm." *IEEE Access* 11: 7829–36.

[9] Jiang, Weihua, Siyuan Zheng, Yuehai Zhou, F Tong, and Ryan Kastner. 2018. "Exploiting Time Varying Sparsity for Underwater Acoustic Communication via Dynamic Compressed Sensing." *The Journal of the Acoustical Society of America* 143 (6): 3997–4007.

[10] Khan, Md Rizwan, Bikramaditya Das, and Bibhuti Bhusan Pati. 2020. "Channel Estimation Strategies for Underwater Acoustic (UWA) Communication: An Overview." *Journal of the Franklin Institute* 357 (11): 7229–65.

[11] Li, Zhengnan, Diego A Cuji, Yukihiro Kida, Mitsuyasu Deguchi, Takuya Shimura, and Milica Stojanovic. 2023. "Frequency Offset Estimation for High Data Rate Acoustic MIMO-OFDM Systems." In *OCEANS 2023- Limerick*, 1–4. IEEE.

[12] Lu, Dongxue, Guiling Sun, Zhouzhou Li, and Shijie Wang. 2019. "Improved CoSaMP Reconstruction Algorithm Based on Residual Update." *Journal of Computer and Communications* 7 (06): 6.

[13] Mechery, Jesmy J, and M Remadevi. 2017. "Compressive Sensing Based Underwater Channel Estimation." *Procedia Computer Science* 115: 683–90.

[14] Natarajan, Balas Kausik. 1995. "Sparse Approximate Solutions to Linear Systems." *SIAM Journal on Computing* 24 (2): 227–34.

[15] Needell, Deanna, and Joel A Tropp. 2009. "CoSaMP: Iterative Signal Recovery from Incomplete and Inaccurate Samples." *Applied and Computational Harmonic Analysis* 26 (3): 301–21.

[16] Qarabaqi, Parastoo, and Milica Stojanovic. 2013. "Statistical Characterization and Computationally Efficient Modeling of a Class of Underwater Acoustic Communication Channels." *IEEE Journal of Oceanic Engineering* 38 (4): 701–17.

[17] Qiao, Gang, Qingjun Song, Lu Ma, Songzuo Liu, Zongxin Sun, and Shuwei Gan. 2018. "Sparse Bayesian Learning for Channel Estimation in Time-Varying Underwater Acoustic OFDM Communication." *IEEE Access* 6: 56675–84.

[18] Ramirez, Carlos, Vladik Kreinovich, and Miguel Argaez. 2013. "Why $\ell 1$ Is a Good Approximation to $\ell 0$: A Geometric Explanation."

[19] Rao, K Deergha, and B K Kartheek. 2018. "Comparative Performance Analysis of OMP and SABMP for Massive MIMO OFDM Channel Estimation." In *2018 5th IEEE Uttar Pradesh Section International Conference on Electrical, Electronics and Computer Engineering (UPCON)*, 1–5. IEEE.

[20] Shi, Xiao-Lin, and Yi-Xin Yang. 2016. "Adaptive Sparse Channel Estimation Based on RLS for Underwater Acoustic OFDM Systems." In *2016 Sixth International Conference on Instrumentation & Measurement, Computer, Communication and Control (IMCCC)*, 266–69. IEEE.

[21] Tropp, Joel A. 2006. "Just Relax: Convex Programming Methods for Identifying Sparse Signals in Noise." *IEEE Transactions on Information Theory* 52 (3): 1030–51.

[22] Wu, Xinyu, Chengbing He, Xiangfei Dai, and Yang Zhang. 2020. "An Improved Sparsity Adaptive CoSaMP with Regularization for Underwater Channel Estimation." In *2020 IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC)*, 1–4. IEEE.

[23] Yahia, Asmaa, Onsy Abdel Alim, and Noha Korany. 2023. "Underwater Acoustic Channel Estimation for Pilot- Based OFDM Using Compressed Sensing." In *2023 4th International Conference on Communications, Information, Electronic and Energy Systems (CIEES)*, 1–6. IEEE.

[24] Yang, Zai, Lihua Xie, and Cishen Zhang. 2012. "Bayesian Compressed Sensing with New Sparsity-Inducing Prior." *ArXiv Preprint ArXiv:1208.6464*.