Solitons and the new renewable energy approach

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Abstract - This article aims at deepening research on the new renewable energy production from the dams reservoir. Coupled to the solitary waves, the wind generated waves present an attractive natural phenomenon that deserves to be emphasized. The Adomain numerical method is used to model the practical observation regarding the soliton propagation. The actual cases contained in this document provide an illustration of the given principals.

Keywords - Wind genrated waves; solitary waves; Kd-V equation; Adomian method; dams.

I. CONTEXT PRESENTATION

As mentioned in [1], a new concept of renewable energy generation was presented. The main idea consists of operating the wind generated waves in order to produce an electrical energy. The researchers recall that, based on practical observation of several dams, four steps distinguishing this phenomenon can be identified:

Stagnated water: represents the surface of water in the state of stagnation. No movement is noted at this area.

Wind gusts: transition step in which the wind is blowing in gusts on the surface of water;
Wind generated wave: the surface of water is deformed. Generation of waves due to action of wind on the water is assisted;

Wave propagation: the waves so generated are transported away from their creation zone. This distance is considered important compared to the length of the wind generated wave.
All the above steps are presented schematically on Fig. 1.

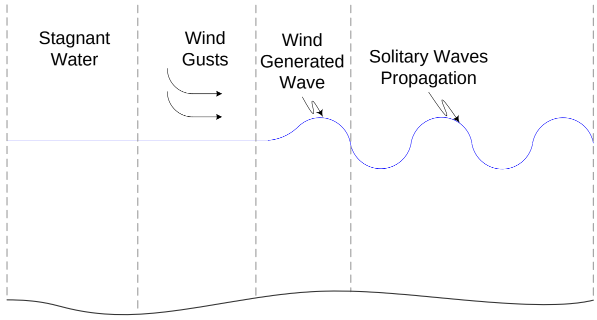

Fig .1. Steps of the phenomenon observed.

Through [1], the researchers were able to establish the physical model governing the creation of the wind-generated waves. In the present document, the researchers will focus more precisely on the fourth step, which consists of wave propagation. The range of propagation of the above waves shows that they can't be qualified as standard ones. As specified on the aforementioned figure, it comes of the solitary wave.

Actually, the solitary wave propagates in nonlinear and dispersive environment. It has localized energy in space and is extremely stable in the presence of disturbances. It moves without changing form or characteristics, and this explains the fact that it is relatively easy to observe in nature, such as in our case.
Hence, the weakly nonlinear Kd-V type theories allow us to elucidate the essential features of these observations. They have the advantage of providing modeling of wave evolution with a reduced wave equation [2].

II. SOLITARY WAVES AND THE KD-V EQUATION

The researchers suggest defining the following, as represented in Fig. 2:

- d: the average depth of the water relative to the free stagnant surface;
- H: the amplitude of the deformation of the surface of water;
- L: longitudinal scale representing a characteristic length of the deformation;
- l: a characteristic width;
- c: a characteristic speed;
- $\mathrm{h(x,y)}$: presenting the water depth relative to the free stagnant surface;
- $\vec{V}(x,y,z,t) = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$: presenting the fluid velocity;
- $\mathrm{p(x,y,z,t)}$: presenting pressure minus the hydrostatic pressure, devised by the density;
- $\mathrm{u(x,y,t)}$: presenting the deformed water elevation relative to the free stagnant surface.

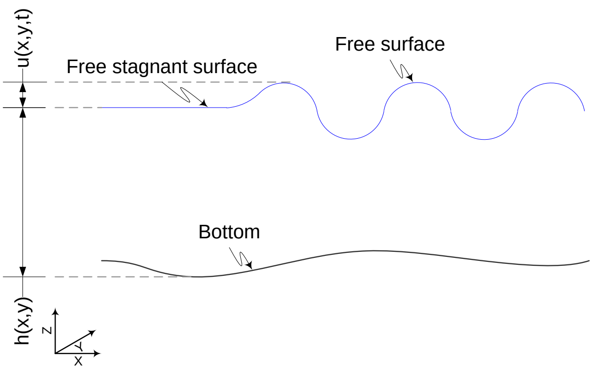

Fig .2. Convention of the characteristic variables of the water deformation.

The fundamental equations (Basic Law of Continuity and Dynamics), constituting the Navier Stokes equations give:

$$\begin{cases} \dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y} + \dfrac{\partial V_z}{\partial z} = 0 \\ \dfrac{d\vec{V}}{dx} + \overrightarrow{\nabla p} = 0 \end{cases}, \qquad (1)$$

with the following boundary conditions:

$$\begin{cases} V_z = \dfrac{du}{dt}, \qquad p = gu \\ V_z + V_x.\dfrac{\partial h}{\partial x} + V_y.\dfrac{\partial h}{\partial y} = 0 \end{cases} \qquad (2)$$

representing conditions respectively at the free surface and the tangent speed at the bottom and with $p = p_n/\rho + gz$, (p_n being the total pressure).

Introducing the following dimensionless variables:

$$x = \lambda.x', \ y = l.y', \ z = d.z', \ t = \frac{\lambda.t'}{c}, \ u = H.u',$$
$$h = d.h', \ V_x = \frac{cH}{d}.V_x', \ V_y = \frac{cH\lambda}{dl}.V_y', \ V_z = \frac{cH}{\lambda}.V_z', \ p = Hgp'.$$

where H is the amplitude, λ is a characteristic length, c is a characteristic velocity, d is the average depth of the water column and l is a characteristic length along the axis y.

For the rest and to simplify editing equations, the researchers rewrite the system of Navier-Stokes equations using dimensionless variables without using « ´», namely:

$$\begin{cases} \dfrac{\partial V_x}{\partial x} + \gamma\dfrac{\partial V_y}{\partial y} + \dfrac{\partial V_z}{\partial z} = 0 \\ \dfrac{\partial V_x}{\partial t} + c_0^2\dfrac{\partial p}{\partial x} + \alpha\left(V_x\dfrac{\partial V_x}{\partial x} + \gamma V_y\dfrac{\partial V_x}{\partial y} + V_z\dfrac{\partial V_x}{\partial z}\right) = 0 \\ \dfrac{\partial V_y}{\partial t} + c_0^2\dfrac{\partial p}{\partial y} + \alpha\left(V_x\dfrac{\partial V_y}{\partial x} + \gamma V_y\dfrac{\partial V_y}{\partial y} + V_z\dfrac{\partial V_y}{\partial z}\right) = 0 \\ \beta\dfrac{\partial V_x}{\partial t} + c_0^2\dfrac{\partial p}{\partial z} + \alpha\beta\left(V_x\dfrac{\partial V_z}{\partial x} + \gamma V_y\dfrac{\partial V_z}{\partial y} + V_z\dfrac{\partial V_z}{\partial z}\right) = 0 \end{cases}, (3)$$

with the following boundary conditions:

$$\begin{cases} V_z = \dfrac{\partial u}{\partial t} + \alpha\left(V_x\dfrac{\partial u}{\partial x} + \gamma V_y\dfrac{\partial u}{\partial y}\right), \quad p = u \ \text{ for } z = \alpha u \\ V_z + V_x\dfrac{\partial h}{\partial x} + \gamma V_y\dfrac{\partial h}{\partial y} = 0 \quad \text{pour } z = -h \end{cases} ; \ (4)$$

and :

$$\alpha = \frac{H}{d}, \ \beta = \frac{d^2}{\lambda^2}, \ c_0^2 = \frac{gd}{c^2}, \ \gamma = \frac{\lambda^2}{l^2}.$$

As part of the previous approximation $H \ll d \ll L$, recovered in the second-order, the researchers deduce that $\beta^2 \ll 1$ and $\alpha\beta \ll 1$.

Thus, they arrive at the Boussinesq system:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial}{\partial x}\left[(\alpha u + h)\widetilde{V_x}\right] + \gamma\dfrac{\partial}{\partial y}\left[(\alpha u + h)\widetilde{V_y}\right] = 0 \\ \dfrac{\partial \widetilde{V_x}}{\partial t} + c_0^2\dfrac{\partial u}{\partial x} + \alpha\left(\widetilde{V_x}\dfrac{\partial \widetilde{V_x}}{\partial x} + \gamma\widetilde{V_y}\dfrac{\partial \widetilde{V_x}}{\partial y}\right) + \dfrac{1}{3}\beta h\dfrac{\partial^3 u}{\partial t^2\,\partial x} = 0 \\ \dfrac{\partial \widetilde{V_y}}{\partial t} + c_0^2\dfrac{\partial u}{\partial y} + \alpha\left(\widetilde{V_x}\dfrac{\partial \widetilde{V_y}}{\partial x} + \gamma\widetilde{V_y}\dfrac{\partial \widetilde{V_y}}{\partial y}\right) + \dfrac{1}{3}\beta h\dfrac{\partial^3 u}{\partial t^2\,\partial y} = 0 \end{cases}; \ (5)$$

with :

$$\widetilde{V_x} = \frac{1}{h}\int_{-h}^{0} V_x.\,dz,\ \ \widetilde{V_y} = \frac{1}{h}\int_{-h}^{0} V_y.\,dz\ . \quad (6)$$

If the researchers focus on solutions $\frac{\partial u}{\partial \tau} + \frac{\partial u}{\partial x} = \ominus(\alpha, \beta, \gamma)$, with $\tau = c_0.t$, for which they accept that solutions spread to the positive x, then they deduce the Kd-V equation for a two-dimensional flow:

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial \tau} + \frac{\partial u}{\partial x} + \frac{3}{2}\alpha u\frac{\partial u}{\partial x} + \frac{1}{6}\beta\frac{\partial^3 u}{\partial x^3}\right) + \frac{\gamma}{2}\frac{\partial^2 u}{\partial y^2} = 0. \quad (7)$$

In the one-dimensional case, either to $\gamma = 0$, the same equation becomes:

$$\frac{\partial u}{\partial \tau} + \left(1 + \frac{3}{2}\alpha\right)u\frac{\partial u}{\partial x} + \frac{1}{6}\beta\frac{\partial^3 u}{\partial x^3} = 0. \quad (8)$$

There exist many variants of the Kd-V equation depending on the case study. In a configuration similar to ours, the above equation can take the following form [2]:

$$\eta_t + (c_0 + \alpha_1\eta + \alpha_2\eta^2)\eta_x + \beta_1\eta_{xxx} = 0, \quad (9)$$

where $\eta(x, t)$ refers to the wave amplitude related to the isopycnal vertical displacement. The coefficients α_1, α_2, and β_1 are functions of the steady background stratification and shear through the linear eigenmode (vertical structure function) of interest. The linear phase speed c_0 is the eigenvalue of the Sturm-Louiville problem for the eigenmode.

The researchers can deduce from the preceding data that handling such equation to get analytic solution cannot be considered as an affordable issue.

The situation can be more complicated when the boundary conditions change due to location and intrinsic site parameters. This is why the researchers present a numerical method in order to insure resolution of the above equation while complying with the efficiency required.

III. THE ADOMIAN DECOMPOSITION METHOD APPLIED TO THE KD-V EQUATION

Consider the Kd-V equation as follows:

$$\frac{\partial u}{\partial t} + 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0,\quad u(x, 0) = f(x) \quad (10)$$

which can be rewritten as follows:

$$\frac{\partial u}{\partial t} = -Ru - 6N(u),\ \ u(x, 0) = f(x) \quad (11)$$

where $R = \partial^3/\partial x^3$ represents the linear operator of the Kd-V equation and $N(u) = u\,\partial u/\partial x$ is the non-linear function. According to the ADM, the solution is expressed by:

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t), \quad (12)$$

and the non-linear part:

$$N(u) = \sum_{n=0}^{\infty} A_n, \quad (13)$$

with:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \quad (14)$$

By integrating with respect to time and using the initial conditions the researchers have:

$$u(x, t) = f(x) - \int_0^t [Lu + 6N(u)]ds. \quad (15)$$

So:

$$u(x,t) = f(x) - \int_0^t \left[L\left(\sum_{n=0}^{\infty} u_n(x,s)\right) + 6\sum_{n=0}^{\infty} A_n \right] ds. \quad (16)$$

For the KdV equation, the Adomian polynomials can be expressed as follows:

$$A_n = \sum_{i=0}^{n} u_i \frac{\partial u_{n-i}}{\partial x}. \quad (17)$$

This allows the researchers to deduce $u_n(x,t)$, namely:

$$\begin{cases} u_0(x,t) = f(x) \\ u_{n+1}(x,t) = -\int_0^t [Ru_n + A_n] ds, \quad n \geq 0 \end{cases} \quad (18)$$

with the following initials conditions:

$$u(x,0) = \frac{1}{2}\operatorname{sech}^2\left(\frac{x}{2}\right). \quad (19)$$

The researchers proceed in the following to compute $A_n(x,t)$ and $u_n(x,t)$ for $n \in [\![0,10]\!]$ and try to determine the approximate solution $\tilde{U}_n(x,t)$ using the following formula:

$$\tilde{U}_n(x,t) = \sum_{i=0}^{n} U_i(x,t) \quad (20)$$

$$= f(x) - \int_0^t \left[L\left(\sum_{i=0}^{n} u_i(x,s)\right) + 6\sum_{i=0}^{n} A_i \right] ds. \quad (21)$$

The numerical studies can be effectively elaborated by the use of this computational method. Once created, the researchers can predict the evolution of the solitary waves with respect to the spatial and time variables. The following figure gives an idea about the numerical calculation obtained [3] and [4]. There exist other numerical methods that can also be used depending on their calculation effectiveness [5] and [6].

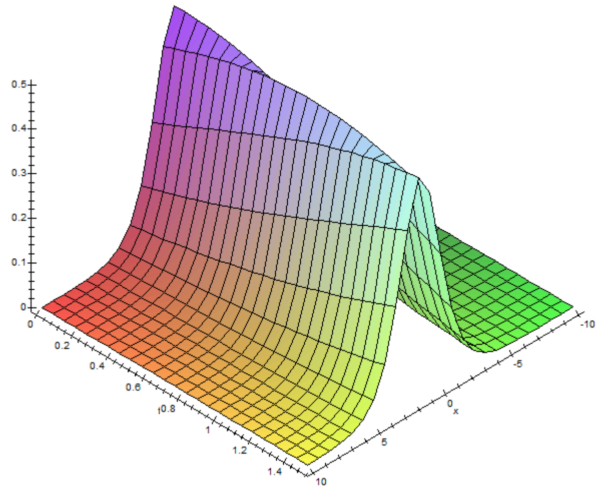

Fig .3. Numerical solution of the Kd-V equation using the ADM method.

IV. INTRINSIC CONDITIONS FAVORING THE EXPLOITATION OF SOLITARY WAVES

The efficiency of the new renewable energy production approach is closely related to the implementation of site conditions. As presented in [7] several parameters are taken in consideration in the site selection. The researchers would like to mention those with critical issues as follows:

- The wind speed constitutes one of the main parameters to take in consideration. The wind speed, the height of the wind generated waves, the swept area and the generated power all together evolve according to the same trend.
- The second one is the wind direction. Indeed, to get optimal conditions of the power extraction, one needs to have both: a regular and a dominant wind direction.

- The third one is the area of the water surface. It allows increasing the recovery of energy during wave propagation.

- Another point is the availability of hydroelectricity plant in order to benefit from the direct injection of the generated electrical power in the network.

Another advantage of this new approach, in addition to those presented previously, lies in benefits due to the intrinsic features of the site implementation. In the following, the researchers will introduce two particular cases illustrating this fact.

A. Topography of the dam location

The speed of the wind is further appreciated in the case of existence of a dominant wind direction. In this sense, the topography of the dam site can filter the wind direction so as to keep a dominant direction. Thus there is consistency in the direction of the wind generated waves.The dam of Ouirgane located in the Atlas Mountains, near of Marrakech city (about 66 Km), reflects this principle. As illustrated in Fig. 4, the dam reservoir along the thalweg.

Fig .4. Satellite image of Ouirgane dam (31°11'23.3"N 8°05'16.5"W) with indication of the wind dominant direction.

Fig. 5 shows that the watersheds elevation constitutes a natural filter with respect to the wind direction: only the wind blowing in a direction parallel to the crest line of the mountains contributes to the creation of the wind generated waves.

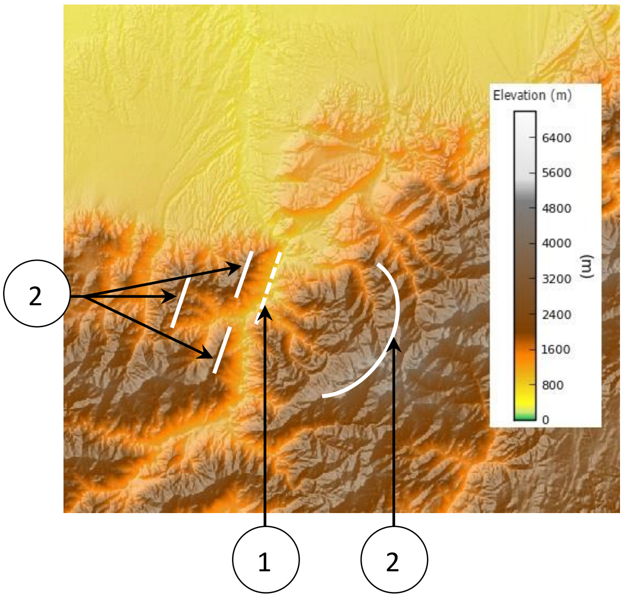

Fig .5. Elevation map at the Ouirgane dam. (1) Refer to the thalweg axis and (2) refer to crest lines of the mountains surrounding the dam reservoir.

Even if the wind direction distribution relating to this region is dispersed among several directions (Cf. Fig. 6), the naturel filter due to topographic feature isolates only the NNE as a dominant wind direction.

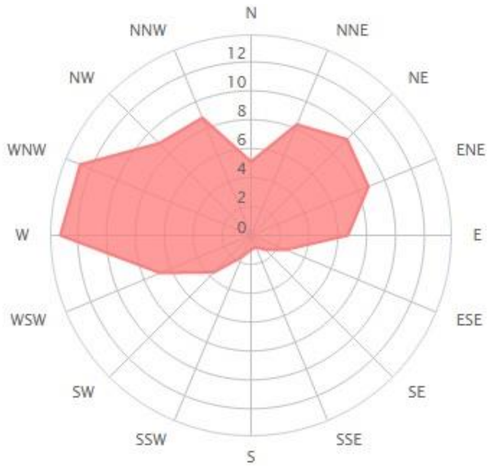

Fig .6. Annual wind direction distribution at Marrakech-Menara airport.

The particular interest reached through this configuration is that the solitary waves once generated continue to propagate along the dam reservoir and do not break. Thus the energy extricated can be recovered a multitude times.

B. Dam wall orientation and solitary waves

We detailed at the beginning of this document the different aspects of the solitary waves, in order to use their application in the new renewable energy concept. The propagation of these waves further away from their emplacement generation offer significant opportunities. This performance can be improved in the case where the dam wall has an orientation perpendicular to the solitary waves propagation as shown in Fig. 7.

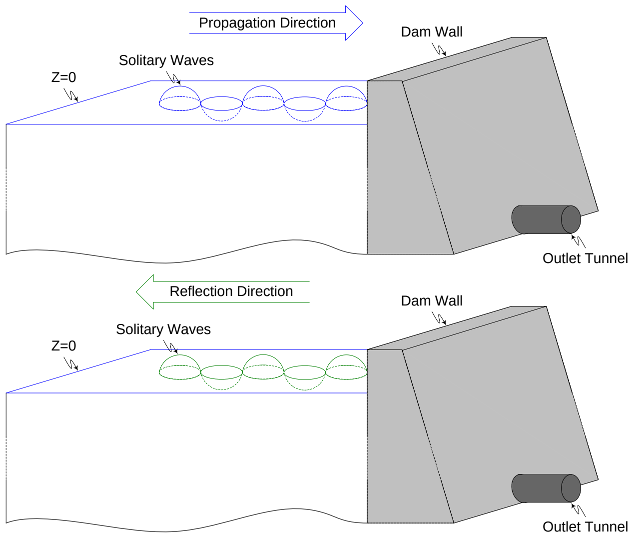

Fig .7. Propagation and reflection of the solitary waves against the dam wall.

For a phenomenon happening a single time, alias the wind generated waves, the swept area is duplicated. This means that the efficiency of any unitary conversion installation responding to this condition will double. The electrical production will occur in the first propagation step and also in the second one after dam wall reflection. Allal Al Fassi dam (about 52 km from Fes city) presented in Fig. 8 gives an appropriate illustration concerning this case. Effectively, as shown in the wind direction distribution in Fig. 9, the dominant wind direction NW is perpendicular to the aforementioned dam wall.

Fig .8. Satellite imagery of Allal Al Fassi dam (33°55'53.0"N 4°40'37.9"W).

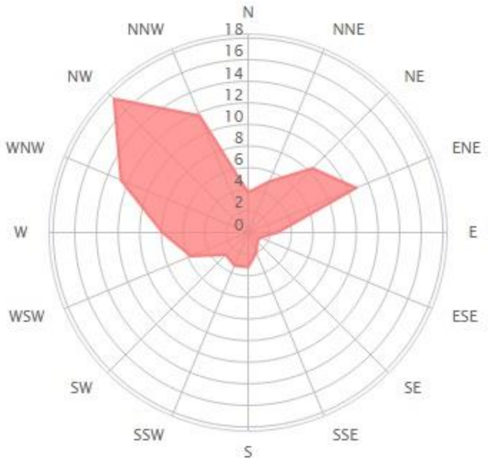

Fig .9. Annual wind direction distribution at Fes-Saïss airport.

V. CONCLUSIONS

In this paper the effect of the combination of the wind generated waves and the solitary waves was proven. Once created under the particular conditions defined, the wind generated waves are transported away from their creation area via solitons. Based on this, the swept area increases significantly and the propagation waves trajectory in dams reservoir becomes longer.

Taking in consideration two particular cases, favorable topographical dam location and the well dam wall orientation, the above performance is further enhanced.

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