Modernization of old mooring structures (case study)

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ABSTRACT

Quay wall is one of the key elements in harbor facilities and equipment, this type of quay wall has been popular in construction of moorings and ships in transportation industry as well as importation and exportation. Due to the demanding big amount of investment in port structures, the construction of a quay wall becomes more important day by day.

During the last four decades, the call for more effective port operation stimulated the development of new cargo handling methods and new cargo handling and hauling equipment, and resulted in dramatic changes in ship size and shape, which have direct impact on the port and harbor development (Carl A, Thoresen, 2007, Port designer's handbook). The advance technology in transporting commodities by water transportation results in a new approach to planning, design and modernization of existing ports.

In general, modernization of the existing facility is aimed at increasing the depth of water in front of the existing wharf, increasing the load carrying capacity of the structure, or both. The techniques used to achieve these goals vary depending on foundation geotechnical parameters, type and physical condition of the existing structure, and operational conditions (Handbook of Port and Harbor Engineering, page 917).

Gravity-type structures are those that develop their resistance to soil pressure and miscellaneous loads primarily from their own weight. Various kinds of gravity-type structure are used (Port Elements: Design Principles and considerations, page 142). The advantages of these quay walls type are easy construction technology, good durability and preferred costs (M. Shafieefar, A.R. Mirjalili, 2014).

This paper is searching to reach an optimum economic solution for improving the stability of the existing wharf. Alexandria port consists of 75 quays, 73 of them are gravity wall type (concrete block type).

Focusing, on studding Seven main Scenarios and 57 cases of relieving structure had been studied.

All these cases were studied using a specialist physical model to simulate the real problem. MIDAS GTS NX foundation, which implemented in this research, is a finite element program for many application including geotechnical applications and foundation engineering in which physical models are used to simulate the problem to reach an optimum construction type by studying the change in the stresses below the bearing concrete block of the quay.

Finally, the seven scenarios had been studied and at last the anchored pile solution is the most optimum solution and the stress result under the bearing block had been improved comparing with the initial case which shows the decrease and re-distribution of the stresses under the bearing block by a percentage about 22 % and two improvement factor 1.5 & 2.0 from the initial load had been studied using Midas Program to calculate the improvement in Live Loads due to increasing in contact stresses. The result shows that the Front Quay Live Load had been improved from 40 Mpa to 80 Mpa, Pallor load from 3 t/m to 6t/m, Crane load from 450 Mpa to 900 Mpa and the Back yard from 40 Mpa to 80 Mpa.

1- Introduction:

There are three main types of quay walls the first type is the Gravity Quay Walls (Block Type Quay Walls, Caisson Walls & L walls) the second type is the Embedded Walls (Cantilever Walls & Anchored Walls) and the third type is the Open Berth Quays (Retaining Wall & With Embankment) (Karakuş, Hülya Ph.D., Department of Civil Engineering , 2013).

In Alexandria Port, the Concrete Blocks Quays (Gravity Walls type) are the common type of quay. Alexandria port consists of 75 quays, 74 quays of them are gravity wall type and the last one is embedded wall type, which can consider the oldest type of berth structures (Carl A, Thoresen, 2007, Port designer’s handbook). That is because of their durability of construction, economic in cost and the possibility to reach a deep seabed level. Gravity structures can be used when seabed is of good geotechnical conditions (like stone, compact sand or stiff clay). (BS: 6349(1988), Part 2: Design of Quay wall).

Gravity quay walls are designed for three main criteria; sliding, overturning and allowable bearing stress under the base of quay wall (M. Shafieefar, A.R. Mirjalili, 2014). Two types of forces act on the gravity retaining quay wall, namely: stability forces and failure forces.

The stability of the wall may be quite sensitive to many factors; depth of the wall, pulling force, back filling and soil characteristics.

Effect of different factors on the stability needs to be investigated. The study focuses on the analysis of the stability of existing gravity concrete blocks quay wall and the cases of re-functionality of the quay; and obtaining an optimum solution for the re-stability of existing old quays in Alexandria ports.

2- Relieving System

Modernization of gravity type wall using the relieving system to increase wall's load-carrying capacity (soil pressure relieving systems) and can be classified as one of the main four basics methods as follows:

a- Soil Grouting

b- Piled System

c- Soil Replacement

d- Piled Platforms for quay wall

Combination of both techniques can be used (Handbook of Port and Harbor Engineering, page 920.).

3- Case study

The Port of Dekheila was constructed as a natural extension of Alexandria port and are numbered from quay no. 90 tell quay no. 98. The study is focusing on quay no. 96 which has a length of 1004 meter, 14 m depth and the construction type is a gravity quay (Concrete block type) and was constructed as a general cargo quay at 1990 then been used as a container terminal at 1999 as shown in figure 2 El-Dekheila Port.

3-1 The existing quay condition

Container terminal in Dekheila port quay no. 96 as shown in figure 1 had been constructed at 1985. The construction type of the quay is concrete block wall which consist of six main blocks ended by a concrete cap as shown in figure 2 (Initial case) which explain the cross section of the quay by real dimensions at Location A in the map at figure 1.

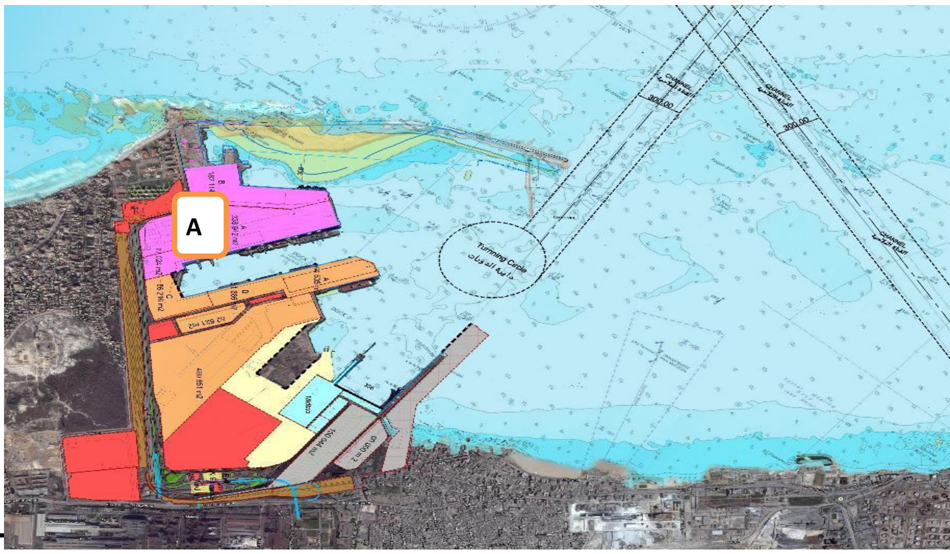


Figure 1 Port of Dekheila Container Terminal Quay 96 locations A

Type	Φ (Angle of friction)	K_a	δ_{dry}	δ_{sub}
Sand	30	0.333	1.8	0.8
Backfill	40	0.217	1.8	0.8
Average	36.67	0.252	1.8	0.8

The quay had been constructed at 1991 as a general cargo berth and had been used as a container terminal. In 2016 admistrators of the container terminal decide to develop the terminal to berth the container C Class vessels by increasing the quay depth from -14 m to -17 m and change the specification of the STS gantry cranes. Studying the development process of the quay to increase the depth and life loads of the terminal, first we must to study the existing condition of the stability of the quay and increase the stability if needed.

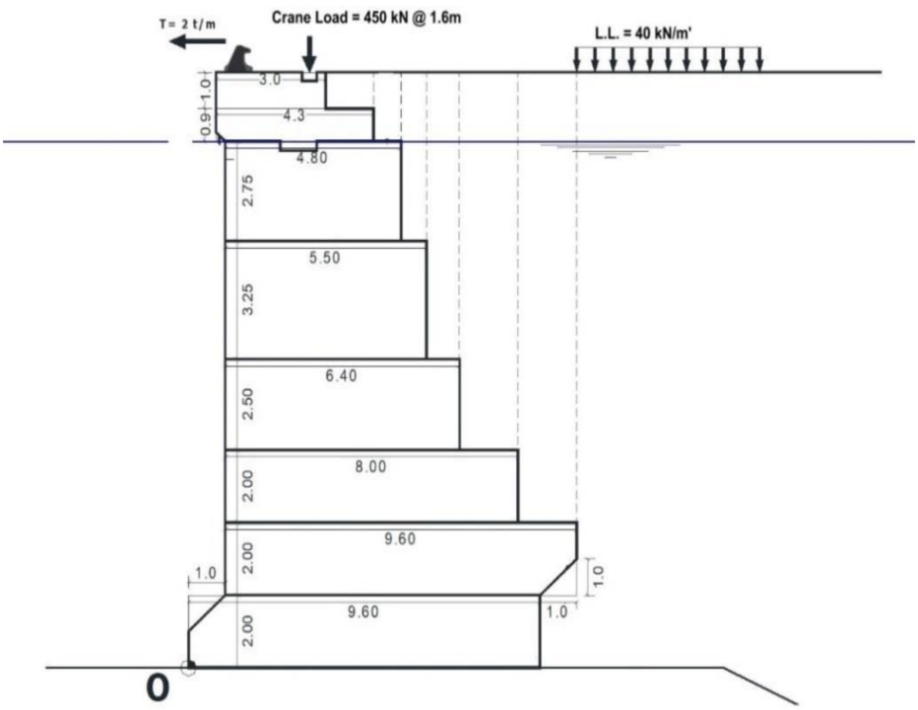


Figure 2 Geometrical profile of Quay 96 with dimension

3-2 Studying Quay Stability

Calculating the stability of a gravity type quay wall, the following items should be examined in general: (MIRJALILI, M, 2004)

1. Sliding
2. Overturning
3. Bearing capacity of the foundation
4. Circular slip

The stability of the gravity wall (concrete block type) may be quite sensitive to many factors; depth of the wall, pulling force, soil characteristics will be consider (B. El-Sharnouby et al./ Gravity quay walls, 2004.). Every retaining wall supports a "wedge" of soil. The wedge is defined as the soil which extends beyond the failure plane of the soil type present at the retaining wall site, and can be calculated once the soil friction angle is known.

Calculating the stresses under the bearing block of the quay to the base layer had been concluded as the initial case of the study using three different programs Midas GTS, Abaqus and Plaxis 8.5.

4- Numerical study and Verification

Calculating the initial case for the marine gravity wall by using several Numerical models. The first numerical model by using Abaqus and the second by using Midas GTS. Comparing the two results as shown on figure 3 the Abaqus and the Midas results curve has the same trend.

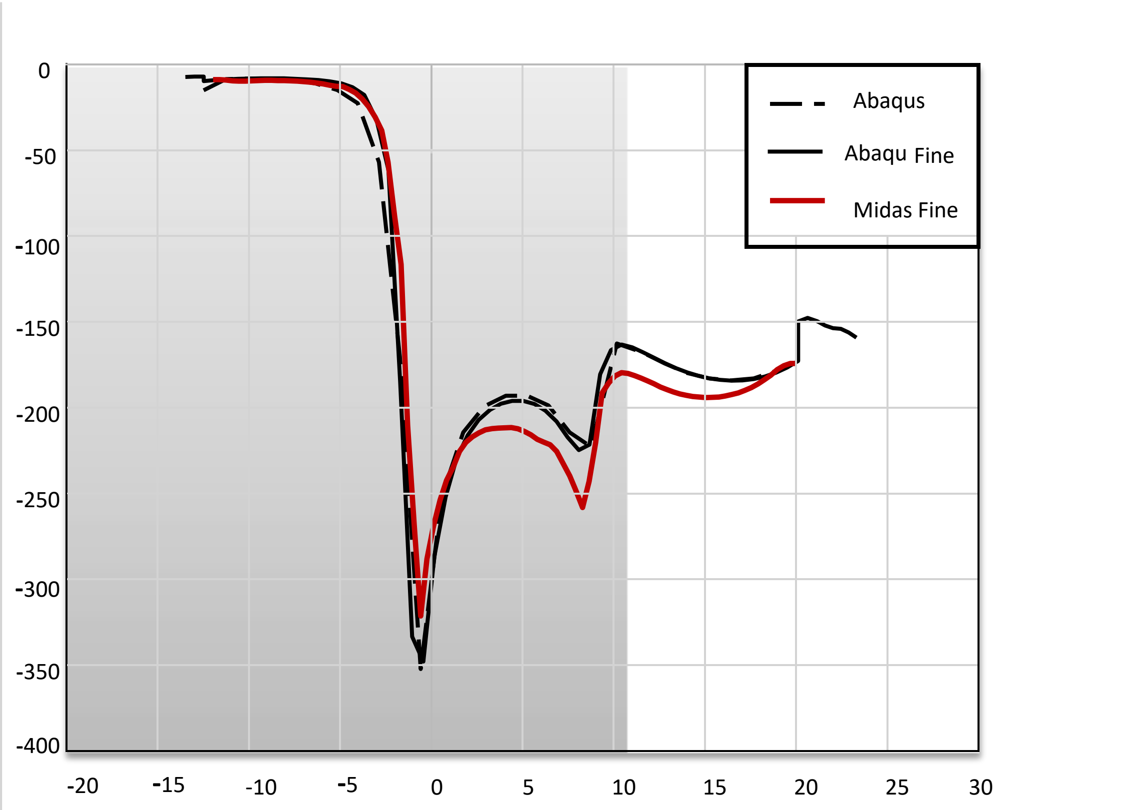


Figure 3 Comparison of stress under the bearing blocks for the initial case with several models

The hard soil model (HSM), used in the Midas GTS software is primarily used for hard soils such as gravels, sands and heavily over-consolidated cohesive soils. This is mainly because the HSM was developed on the assumption the plastic straining is dominated by shearing and associated volumetric strains are relatively small and cause dilation rather than compacting, which is a property of noncohesive and heavily consolidated cohesive soils. In contrast to this basic formulation of the model, (Freiseder (1998)) believed that the HSM give more realistic results on deformation of the wall and settlement of ground behind the wall in an excavation in such as gravels, sands and normally consolidated clay than the other models such as The Mohr-Coulomb Model (MCM), which is an elastic-perfect plastic model. Only the Mohr-coulomb Model (MCM) is available in the software package ABAQUS.

5- The Model Description:

Midas GTS NX is a comprehensive finite element analysis software package that is equipped to handle the entire range of geotechnical design applications including deep foundations, excavations, complex tunnel systems, seepage analysis, consolidation analysis, and embankment design, dynamic and slope stability analysis. GTS NX also has an advanced user friendly modeling platform that enables unmatched levels of precision and efficiency.

As the next generation geotechnical analysis software, Midas GTS NX features the newest development in cutting-edge computer graphics and analysis technology. GTS NX fully supports the latest 64-bit OS Graphic user interface. The intuitive interface will enable new users to easily integrate the software in their work process. The fast analysis speed, outstanding graphics, and output capabilities will provide users with a new and advanced level of geotechnical design.

6- Boundary Condition

Midas automatically imposes a set of general fixities to the boundaries of the geometry model. These conditions are generated according to the following rules:

- The ground surface of the domain is free in all direction X_x , $U_{z,\text{ and}}$ $U_y \neq 0$).
- Vertical boundaries of the domain with their normal neither in x-direction nor in z-direction are fixed that is to say in x-direction and z-direction ($U_x = U_z = 0$) and free in y-direction, ($U_y \neq 0$).
- The bottom boundary of the domain is fixed in all direction ($U_x = U_y = U_z = 0$).
- The dimensions of the boundary of the case study are the width is 10.0 m while the depth is 36.0m (14.0 m below water level, 2.0 m above water level and 20 m in the soil ground).

7- Scenario, Results and Discussion:

7-1 Initial case

Using Midas program to solve the initial case and the seven scenarios contain 57 different cases with different x, d, phi for each scenario.

The seven scenarios of relieving structures are as shown:

1- The initial case (concrete block wall) the case study case

- Calculating the stresses below the bearing block and the vertical, horizontal displacement (as shown in figure in figure 4).

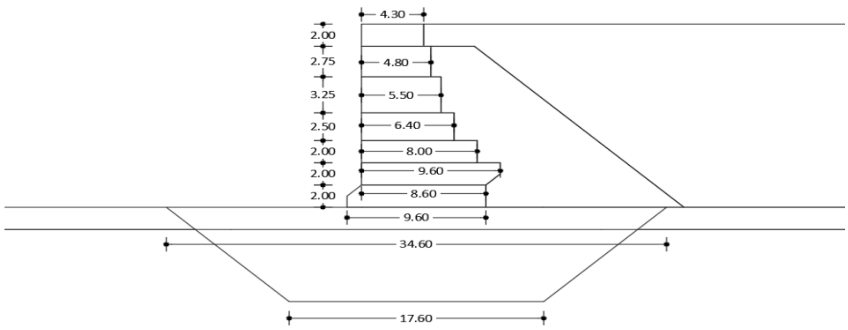


Figure 4 Initial Case

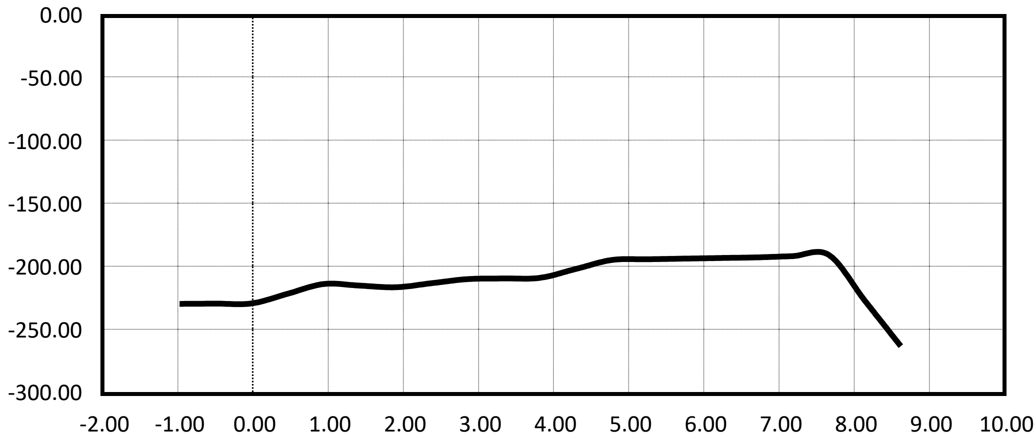


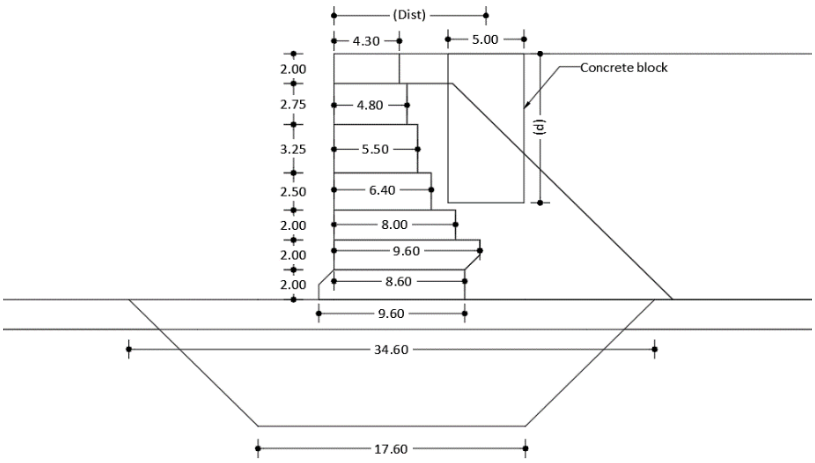
Figure 5 stresses under the bearing block of the initial case

The two axis of the comparison are the vertical axis indicate to stresses under the bearing block according to the initial case of the existing case study and the horizontal axis is the length of the bearing block. The result stresses will be the original curve as shown in figure 5 to study the improvement in the contact stresses due to the cases studied of relieving structure to select the more effective case.

7-2 Concrete block relieving structure

The 1st Scenario is the Concrete Block relieving structure

This scenario contain six cases with different distance and depth for the relieving structure as shown in the table below and figure 6 to obtain the more effective solution.



(Dist)	(d)	Model Name
10	5	BL 1
10	7.5	BL 2
10	10	BL 3
15	5	BL 4
15	7.5	BL 5
15	10	BL 6

Figure 6 Concrete Block relieving structure

Evaluating six models for relieving concrete block structure in the active wedge from Bl1 to Bl6, after checking the results shows that the black curve is the best result BL(3) as shown in figure 7 which is Concrete Block with 10m depth and 10 m Distance from the front of the quay.

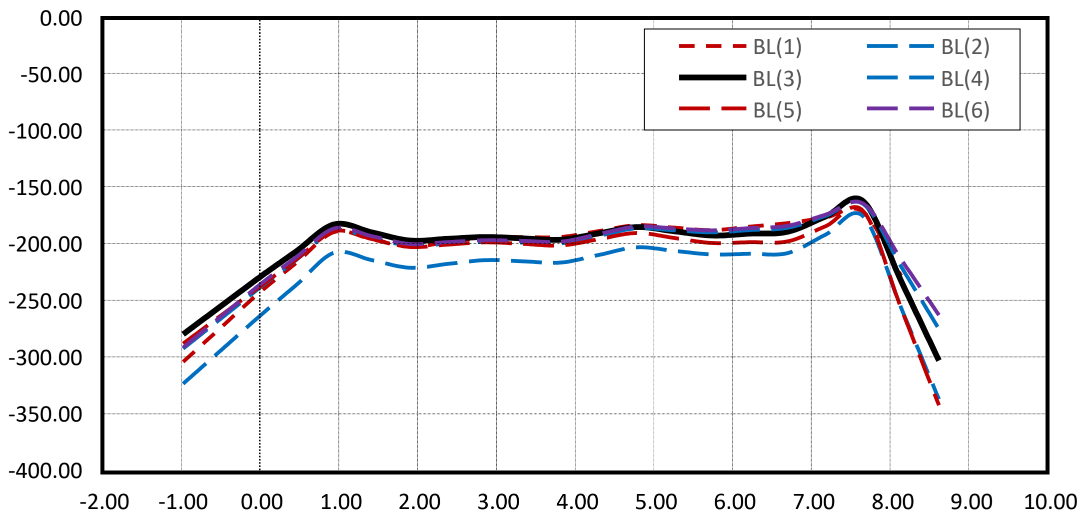
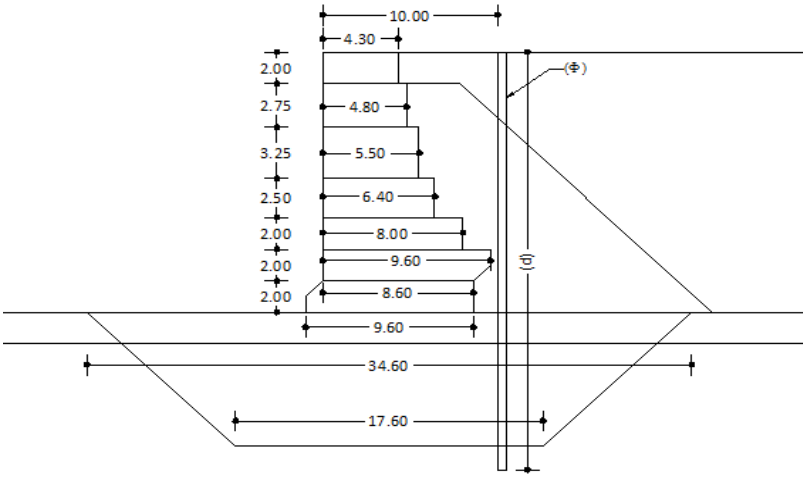


Figure 7 Stresses under the bearing block due to several cases of concrete block relieving structure

7.3. Single Row Pile relieving structure

The 2nd Scenario is the single row piles relieving structure

This scenario contains fifteen cases named from Pl1 to Pl15 for different types of relieving supporting structures in the active wedge as shown in the table below and figure 8 with different dimension.



(S)	(F)	(d)	Model Name
Spaced at 2 phi	0.5	16.5	PL(1)
	0.5	21.5	PL(2)
	0.5	26.5	PL(3)
	0.75	16.5	PL(4)
	0.75	21.5	PL(5)
	0.75	26.5	PL(6)
	1	16.5	PL(7)
	1	21.5	PL(8)
	1	26.5	PL(9)
Spaced at 3 phi	0.5	16.5	PL(10)
	0.5	21.5	PL(11)
	0.5	26.5	PL(12)
	1	16.5	PL(13)
	1	21.5	PL(14)
	1	26.5	PL(15)

Figure 8 Single row piles relieving structure

Evaluating fifteen models named from Pl1 to Pl15 for different types of relieving supporting structures in the active wedge with different diameters phi 0.5 , 0.75 & 1.0 m and different depth 16.5 , 21.5 & 26.5 m (from Pl1 to Pl9 the spacing between piles 2 S and from Pl10 to Pl15 the spacing is 3 S) , after checking the results, the black curve is the best result Pl11 shown in figure 9 is the One row Pile with 21.5 meters depth, 0.5 meter Diameter and 3 Phi spacing is the most effective solution in this case.

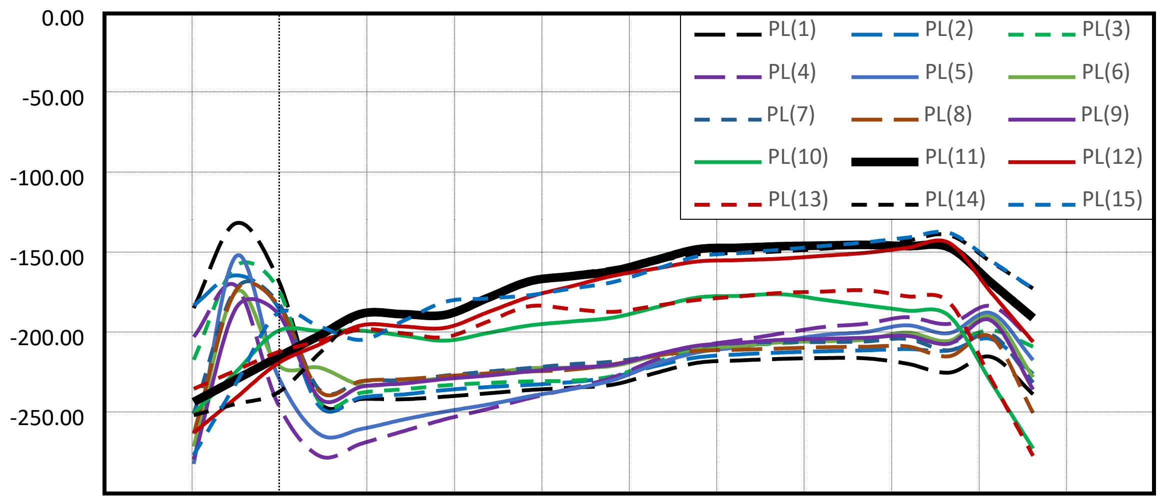


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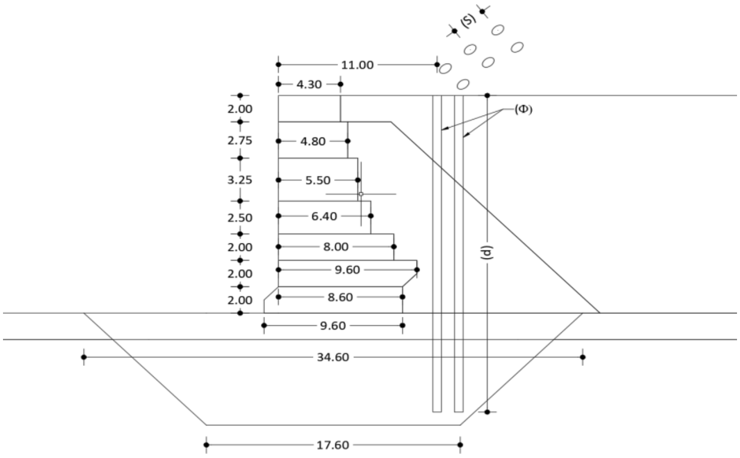
Figure 9 Stresses under the bearing block due to several cases of single row piles relieving structure

7-4 Double Row Piles Relieving Structure

The 3rd Scenario is double row Pile relieving structure.

This scenario contain nine cases with variables diameter phi 0.5, 0.75 & 1.0 m as shown in table below and figure 10 to obtain the more effective solution.



(F)	(d)	(S)	Model Name
1	16.50	2.00	P2R(1)
1	21.50	3.00	P2R(2)
1	26.50	4.50	P2R(3)
0.5	16.50	1.00	P2R(4)
0.5	21.50	1.50	P2R(5)
0.5	26.50	2.25	P2R(6)
0.75	16.50	1.50	P2R(7)
0.75	21.50	2.25	P2R(8)
0.75	26.50	3.38	P2R(9)

Figure 10 Double row staggered pile-relieving structure

Evaluating nine models named from P2R1 to P2R9 for different types of relieving supporting structures in the active wedge with different diameters phi 0.5, 0.75 & 1.0 m and different depth 16.5, 21.5 & 26.5 m (double row pile system variable spaced at 2,0 S, 3,0 S & 4.5 S), after checking the results, the black curve is the best result P2R6 shown in figure 11 is the double row pile with 26.5 meters depth, 0.5 meter Diameter phi and 3 S spacing is the most effective solution in this case.

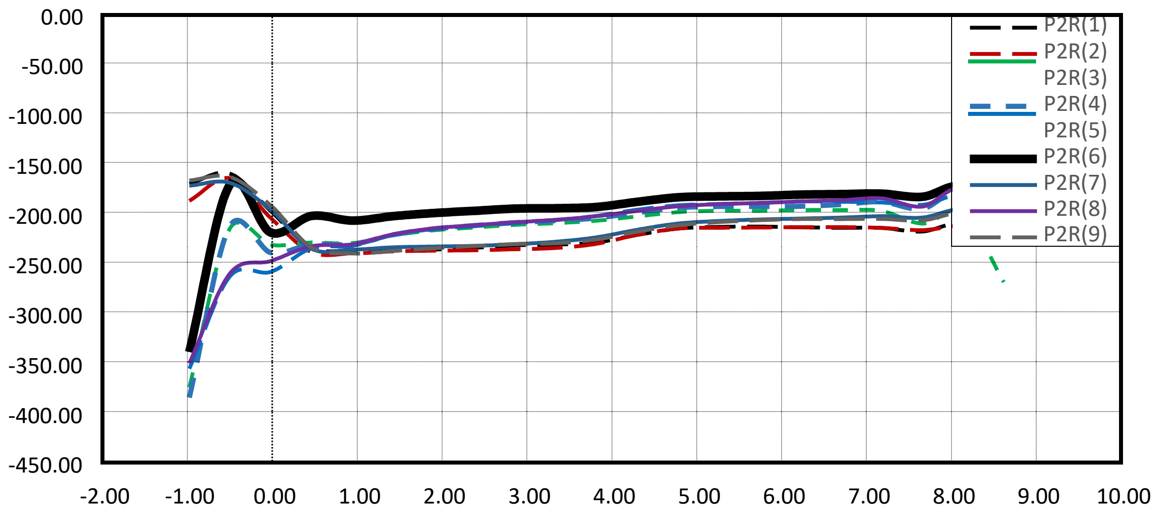
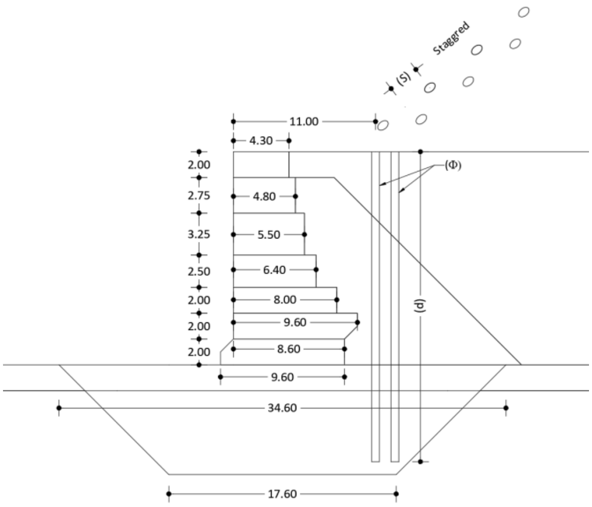


Figure (11) stress under the bearing block due to several cases of double row piles relieving structure

7-5. Double Row Staggered piles Relieving Structure

The 4th Scenario is the double row staggered pile relieving structure.

- This scenario contains nine cases with variable diameter phi 0.5 , 0.75 & 1.0 m and distance between piles as shown in the table below and figure 12 to obtain the more effective solution.



(F)	(d)	(S)	Model Name
1	16.50	2.00	P3R(1)
1	21.50	3.00	P3R(2)
1	26.50	4.50	P3R(3)
0.5	16.50	1.00	P3R(4)
0.5	21.50	1.50	P3R(5)
0.5	26.50	2.25	P3R(6)
0.75	16.50	1.50	P3R(7)
0.75	21.50	2.25	P3R(8)
0.75	26.50	3.38	P3R(9)

Figure 12 double row staggered pile relieving structure

Evaluating nine models named from P3R1 to P3R9 for different types of relieving structures in the active wedge with different diameters 0.5, 0.75, 1,0 m and different depth 16.5, 21.5 & 26.5 m (double row pile system variable spaced at 2Phi, 3 Phi & 4.5Phi), after checking the results, the black curve is the best result P3R6 shown in figure 13 is the double row staggered pile with 26.5 meters depth, 0.5 meter Diameter and 3 Phi spacing is the most effective solution in this case.

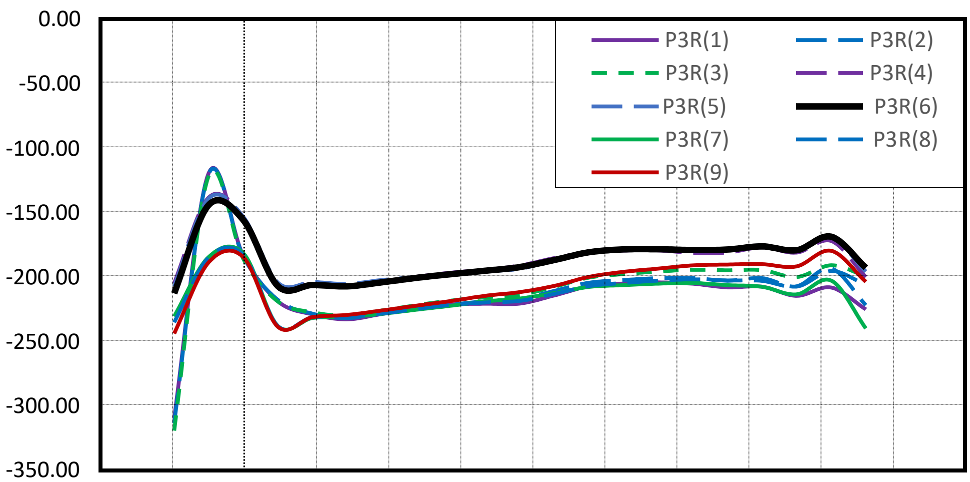


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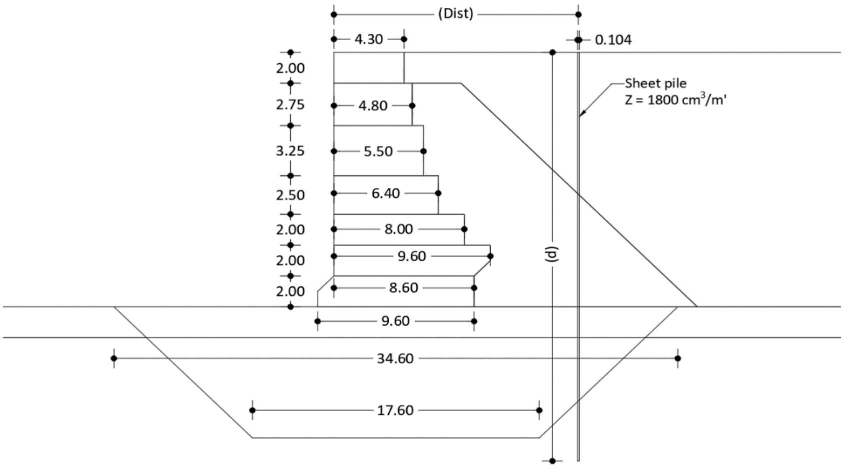
Figure (13) Stresses under the bearing block due to several cases for double row staggered piles

7-6. Single Sheet Pilling Relieving Structure

The 5th Scenario is single sheet pile relieving structure

This scenario contains six cases for single sheet pile relieving supporting structure as shown in table below and figure 14 with variable distance 11 m & 16 m to obtain the more effective solution.



(Dist)	(d)	Model Name
11	16.5	SP 1
11	21.5	SP 2
11	26.5	SP 3
16	16.5	SP 4
16	21.5	SP 5
16	26.5	SP 6

Figure 14 single sheet pile relieving structure

Evaluating six models named from SP1 to SP6 for different types of relieving structures single sheet pile in the active wedge with variable distance 11 & 16 m and different depth 16.5, 21.5 & 26.5m (single sheet pile), after checking the results, the black curve is the best result SP6 shown in Figure (15) is the single sheet pile with 26.5 meters depth, and 16 m distance is the most effective solution in this case.

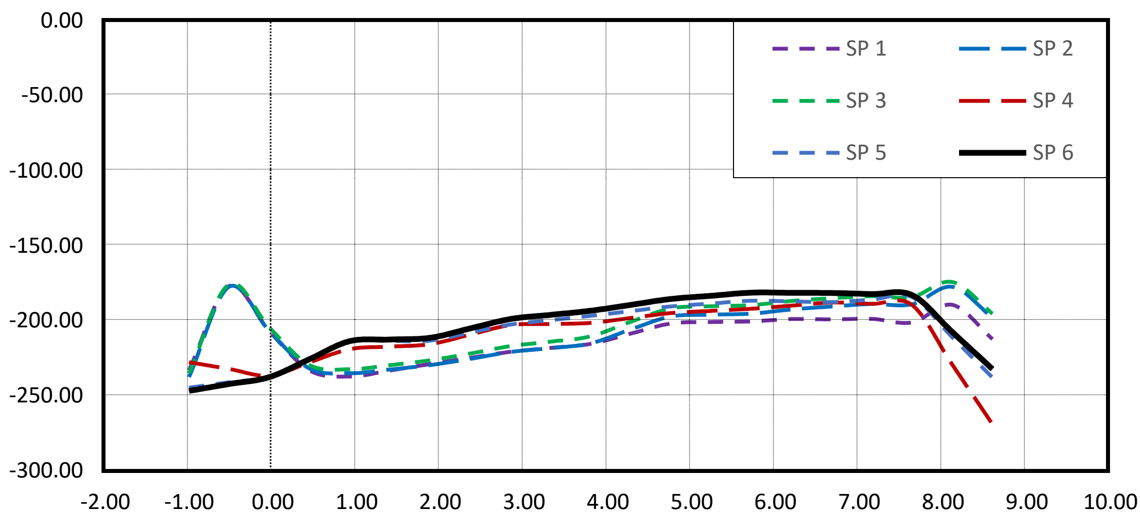
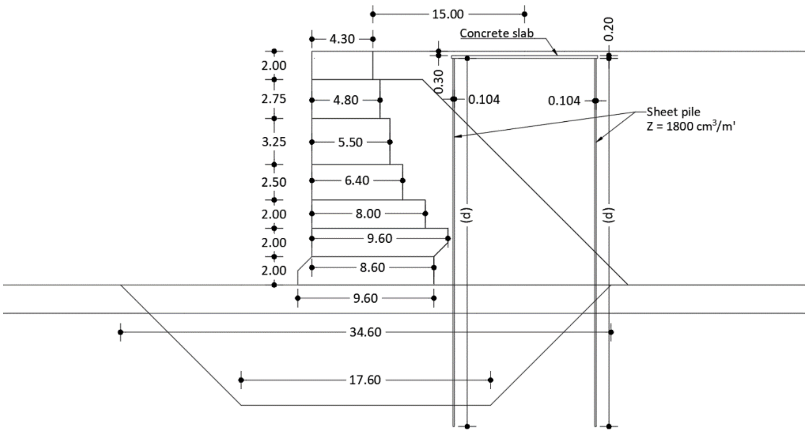


Figure 15 Stresses under the bearing block due to several cases of single sheet piles

7-7. Double Sheet Pilling Relieving Structure

The 6th Scenario is double sheet pile relieving structure

This scenario contains three cases for double sheet piles relieving structure as shown in the table below and figure 16 with constant distance 15m to obtain the more effective solution.



	(d)	Model Name
Distance=15 m	16.5	DSP1
	21.5	DSP2
	26.5	DSP3

Figure 16 double sheet pile relieving structure

Evaluating three models named from DSP1 to DSP3 for different types of relieving supporting structures double sheet pile in the active wedge with constant distance 15m and different depth 16.5, 21.5 & 26.5m (double sheet pile), after checking the results, the black curve is the best result DSP3 shown in figure 17 is the double sheet pile with 26.5 meters depth, and 15 m distance is the most effective solution in this case.

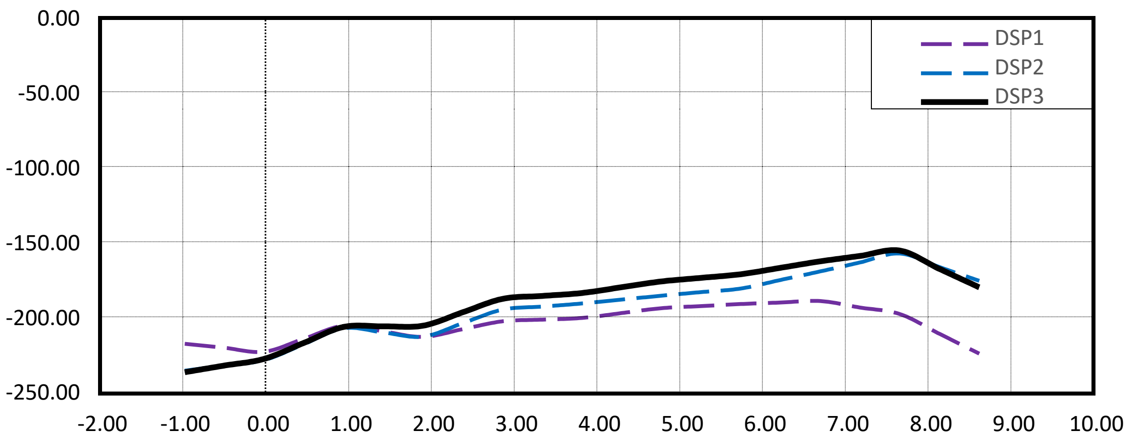
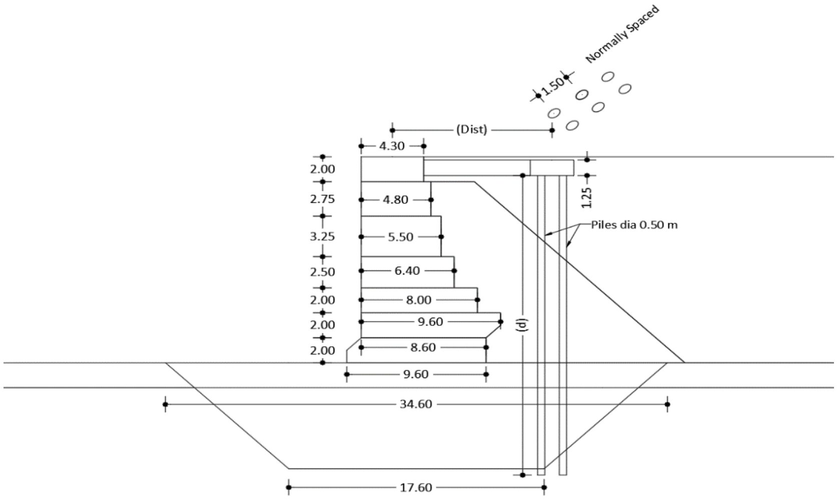


Figure 17 Stresses under the bearing block due to several cases of double sheet piles

7-8. Anchored Piles Relieving Structure

The 7th Scenario is the Anchored pile relieving structure

This scenario contains nine cases for Anchored pile relieving supporting structure as shown in table below and figure 18 with different distance and depth for the relieving structure to obtain the more effective solution.



(Dist)	(d)	Model Name
11	16.5	AP 1
11	21.5	AP 2
11	26.5	AP 3
16	16.5	AP 4
16	21.5	AP 5
16	26.5	AP 6
22	16.5	AP 7
22	21.5	AP 8
22	26.5	AP 9

Figure 18 Anchored pile relieving structure

Evaluating nine models named from AP1 to AP9 for different types of relieving structures for anchored piles in the active wedge with variable distance from the quay 11, 16, & 22 m and different depth 16.5, 21.5 & 26.5 m (Anchored piles is a double row piles diameter 0.5 m with space 3 S and piles depth 26.5 m with a fixed reinforced concrete slab on the quay concrete cap from one side and the piles from the other side with a thickness of 1m), after checking the results, the black curve is the best result AP3 shown in figure 19 is the anchored pile with 26.5 meters depth, 0.5 meter Diameter and 3 Phi spacing at 11 m distance is the most effective solution in this case.

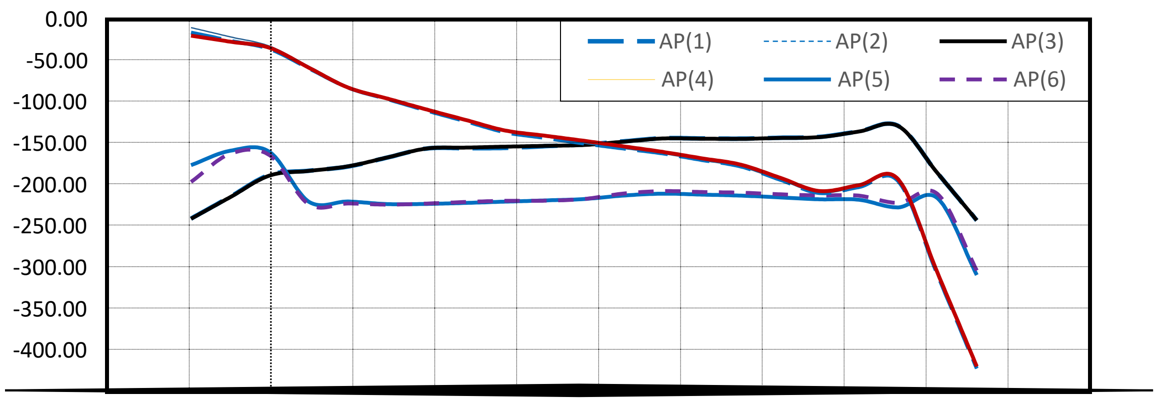


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Figure19 Stresses under the bearing block due to several cases of anchored piles

7-9 Total Comparison between the best solutions of each case

Evaluating all the 7th Scenarios and the 57 cases for different types of relieving structures in the active wedge with variable distance from the front of the quay and different depth after checking the results, the dashed red curve is the best result shown in figure 20 is the anchored pile (Anchored piles is a double row piles diameter 0.5 m with space 3 Phi and piles depth 26.5 with a fixed restated reinforced concrete slab on the quay concrete cap from one side and the piles from the other side with a thickness of 1m), is the most optimum effective solution in all cases.

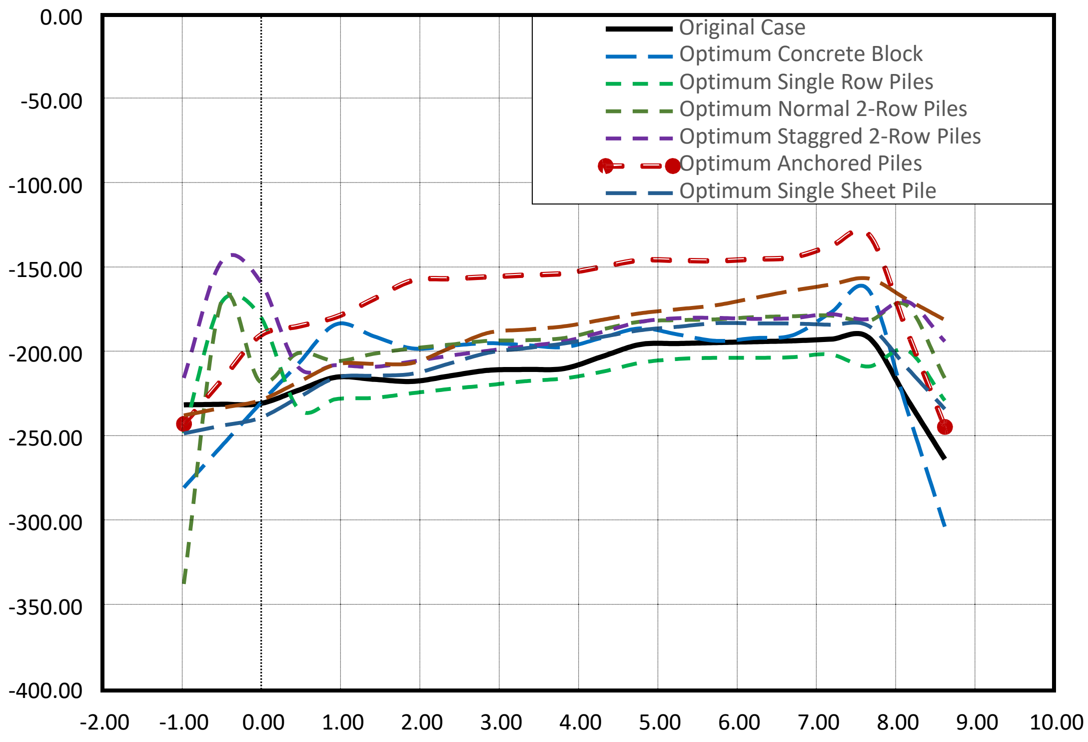

Figure (20) comparison results to stress for the best case for all cases of relieving structure

7-10 comparing the optimum solution with the original case

Comparing the Anchored pile solution relieving structure with the initial case as shown in figure 21 the red curve shows the improvement in the stress due to the anchor pile relieving structure as a solution to decrease the stress under the bearing block by percentage of 22% as shown in figure 22.

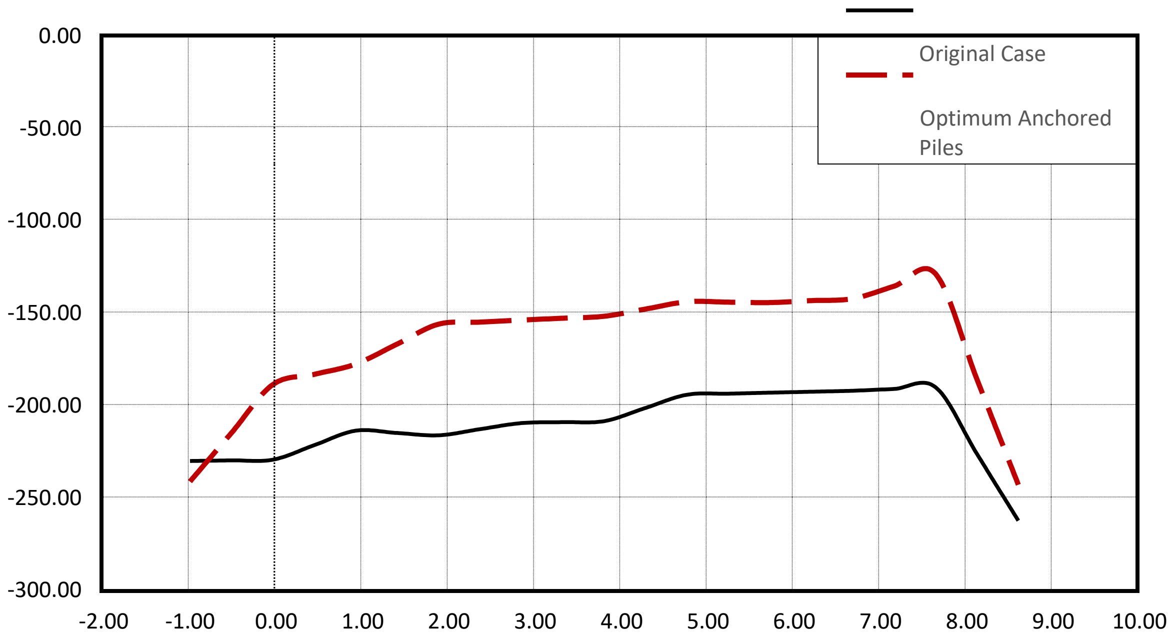

Figure (21) shows the improvement in the contact stress under the bearing block due to relieving structure

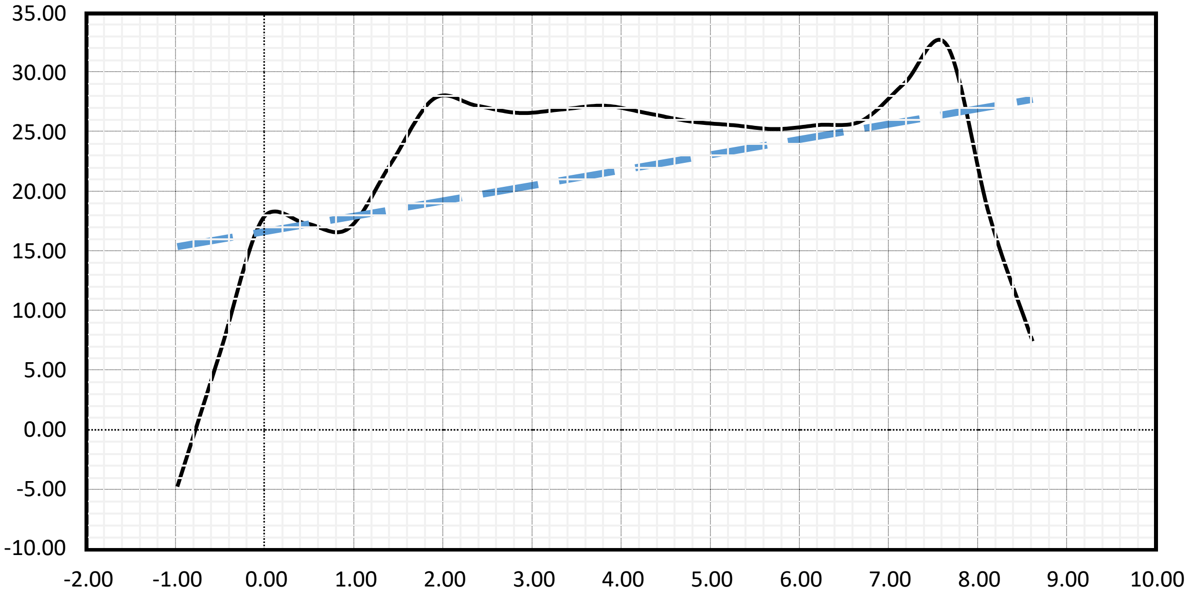

Figure 22 shows the reduction percentage in the contact stress under the bearing block

7-11 Improvement factor (IF) for Loads after improving the contact stresses

Calculating the improvement factor in Live Loads due to improving in contact stresses by a percentage 22% due to the relieving structure. Quay Live Load is 40 Mpa in the initial case two-improvement factor had been studied as shown in the table below and figure 23.

The Load is classified to Polar load, Front Quay LL, Crane Load and Back Yard LL. The initial case loading will be increase with an improvement factor 1.5 and 2 as shown in table below.

The table shows the load cases studied

Case	**Polar load**	**Front Quay LL**	**Crane Load**	**Back Yard LL**
Initial case	3.0 ton	40 Mpa	450 Mpa/1.6 m	40 Mpa
Improvement factor 1.5	4.5 ton	60 Mpa	680 Mpa/1.6m	60 Mpa
Improvement factor 2.0	6.0 ton	80 Mpa	900 Mpa/1.6m	80 Mpa

In figure 23 shows that the improvement factor (IF) 1.5 with the red dashed line and IF 2.0 with the pink dashed line are less in stresses under the bearing block except under the toe which is the f1.

The table shows the value of f1& f2 for all cases.

Case of load	f1 (Kpa)	f2 (Kpa)
Original Case	**230**	**270**
Optimum Anchored	240	240
Improvement Factor 1.5	375	230

Improvement Factor 2.0	425	240

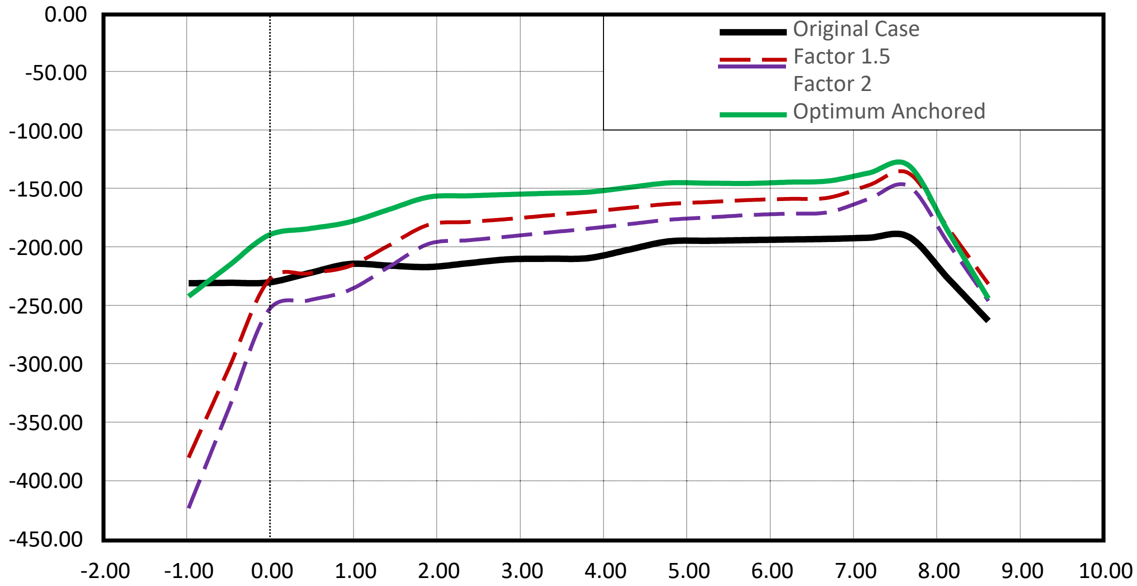

Figure 23 Results of calculating the improvement factor of loads

Studying the vertical displacement under the different improvement factor comparing with the original case.

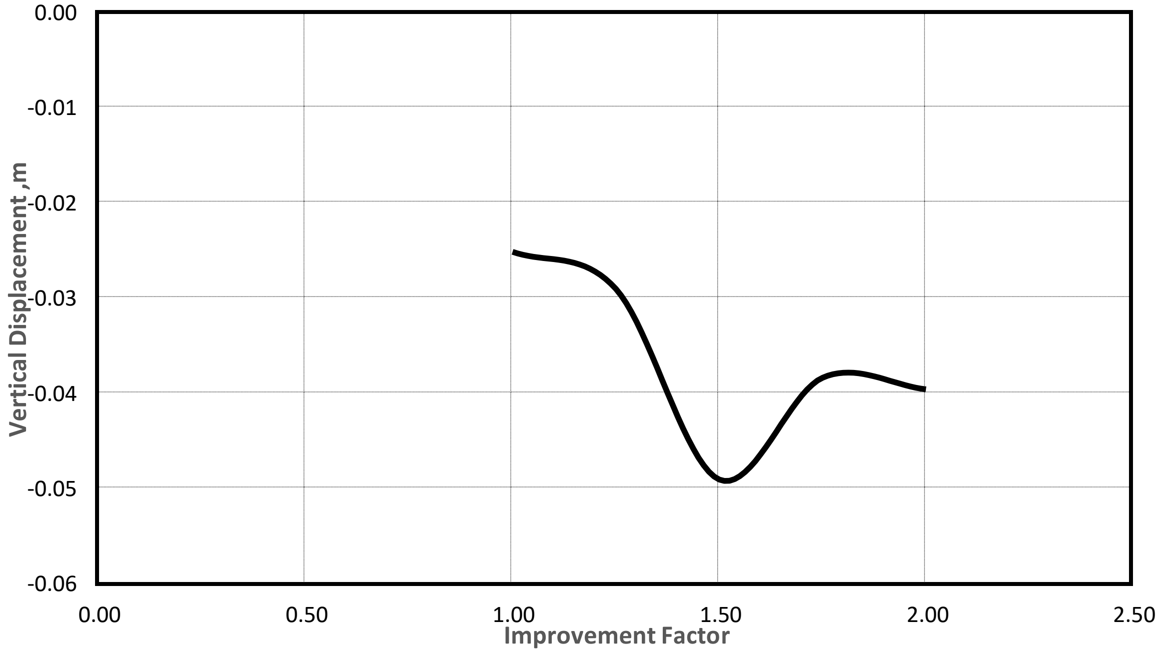

Figure 24 vertical displacements due to improvement factors

The method of improving the ground before exciting the Quay. Due to the great deadweight of the block, block wall berths should be used only on very firm ground in order to avoid settlement. The ground had been dredged to level – 27.0 m and had been replacement by compacted (vibro-compaction) course sand to level – 14.0 m. The structure causes great stresses at the outer edge of the bottom, which is laid on rabble base surface levelled with crushed stone (Carl A, Thoresen, 2002).
The maximum vertical displacement due to the improvement factor of load 1.5 & 2.0 don't exceed 5cm which is excepted according to the (B.S:6349 (1988)

Increasing loads by factorized criteria may led to excessive deformations under the wall , considering the average stresses under the wall not the maximum debating with terzagi's assumptions of flexible foundation .

8- Conclusion

This study developed the performed and efficiency analysis using Midas 3D GTS NX which is a comprehensive finite element analysis software package that is equipped to handle the entire range of geotechnical design applications including deep foundations, excavations, complex tunnel systems, seepage analysis, consolidation analysis, embankment design, dynamic and slope stability analysis. Program with proposed scenarios for several types of relieving supporting structures. Various scenarios of types of structures were proposed; each scenario. The results of this research could help in improving and re-functional of the port quays.

Seven Scenarios through cases had been studied and the optimum solutions had been selected and had been analysis from the stress point of view under the bearing block comparing with the initial case.

Two improvement factors 1.5 & 2.0 from the initial load had been studied using Midas Program to calculate the improvement in Live Loads due to increasing in contact stresses. The result shows that the Front Quay Live Load had been improved from 40 Mpa to 80 Mpa, polar load from 3 t/m to 6t/m, Crane load from 450 Mpa to 900 Mpa and the Back yard from 40 Mpa to 80 Mpa.

Finally, by comparison the seven scenarios the anchored pile solution relieving structure stress result under the bearing block with the initial case shows the improvement in the stress due to the anchor pile structure as a solution to decrease and re-distribution the stress under the bearing block by an average percentage about 22 %.for the concrete block type.

Future work can be done to study the optimum solution to deep the depth of the case study quay and studding the effect of the seismic condition.

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